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Artificial neural network approach for parameter estimation of exponentially damped sinusoids using linear prediction

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Abstract

The paper presents a neural net-based scheme embodying linear prediction techniques and the SVD algorithm to estimate the parameters of exponentially damped sinusoids satisfactorily under low SNR conditions. In the method proposed, a three-layer feed-forward neural network is employed at the output of the SVD block for suppressing bias in the estimated singular values due to the presence of noise. The ANN block is used to keep the singular values constant at their noiseless counterpart, even at SNR less than 0 dB. The method is considered to be the most efficient for parameter estimation at very low SNR.

Keywords: Singular value decomposition, artificial neural networks, linear predictive coding, backward linear prediction.

1. Introduction

Parameter estimation of an exponentially damped sinusoidal signal has been studied rigorously over the past decade. A good number of techniques for the same have been reported in the literature[1–7]. Kumaresan *et al*[1–4]., Reddy *et al*[5]. and Rahman *et al*[7]. have proposed different powerful methods for parameter estimation related to linear predictive coding (LPC). The exponentially damped sinusoidal signals defined over a finite time interval can be completely characterized by the numerator and denominator polynomial coefficients of a transfer function model of large order[8,9]. The linear prediction coding method, proposed by Kumaresan and Tufts (KT), an improved variant of Prony's method[10], is used for the estimation of the coefficients of the transfer function[1,6,8].

The features of the KT method are ,

(i) it uses an overdetermined set of linear equations,
(ii) it overestimates the order of the assumed linear model,
(iii) it uses singular value decomposition to solve the linear set of equations (Wiener–Hopf equations),
(iv) it estimates the backward predictor polynomial coefficients so as to separate the signal poles from that of the noise poles.

The frequency, damping, phase and amplitude of the signal are estimated from the roots of the denominator polynomial and the residues of the transfer function. N signal samples, which are processed to estimate the parameters, decide its robustness. However, the performance of Kumaresan–Tufts method deteriorates when the noise power level is increased. The concept of parameter estimation is based on the distribution of zeros of the prediction error filter that does not prove to be a viable solution in many noisy environments encountered in the power system and signal processing problems such as speech, sonar, communication and radar systems.

The SVD algorithm augments the estimation capabilities of linear prediction at low SNRs. It is found that the distribution of the signal zeros behaves erratically in the Z-plane, when the SNR of the input signal is less than 20 dB. Hence, at low SNR, *i.e.*, below 20 dB, the bias in the frequency is small but is significant in the damping factors. Greater the magnitude of damping more is the bias. There are very few published works on the bias compensation methods[1,2,11] to improve the estimates. Research is being carried out to develop a robust algorithm to exhibit satisfactory performance in different practical environments with low SNR. In this paper, a neural network-based parameter estimation approach is proposed to improve the estimates at low SNR conditions. This scheme employs widely used Kumaresan–Tufts method and an artificial neural network (ANN) to give much better performance at very low SNR. The algorithm employed for adaptation of the weights of the multi-layered network is the widely[12–15] used backpropagation algorithm with sigmoidal activation function.

2. Linear predictive coding algorithm

The mathematical model of the signal can be represented as,

$$y(n) = \sum_{k=1}^{M} a_k e^{s_k n} + w(n);\ n = 1, 2, \ldots, N \tag{1}$$

where $s_k = -\alpha_k + 2\pi f_k$, $k = 1, 2, \ldots, M$ with positive damping coefficients α_k's and pole frequencies f_k s; a_k, $k = 1, 2, \ldots, M$ are the amplitudes, and $w(n)$ is the white Gaussian noise.

The KT method is described using the following procedure.

Step 1

Define the backward linear prediction data matrix A, and vector h for a filter of order L, such that $M \leq L \leq N\text{–}M$, where

$$A = \begin{bmatrix} y(2) & y(3) & \cdots & y(L+1) \\ y(3) & y(4) & \cdots & y(L+2) \\ \vdots & & \vdots & \vdots \\ y(N-L+1) & y(N-L+2) & \cdots & y(N) \end{bmatrix}$$

$$\text{and } h = \begin{bmatrix} y(1) \\ y(2) \\ " \\ " \\ " \\ " \\ y(N-L) \end{bmatrix} \tag{2}$$

Step 2

Find the singular-value decomposition of the matrix A:

$$A = U\Sigma V^H \tag{3}$$

where U and V are the left and right singular vectors of A, respectively, Σ is a diagonal matrix with M largest singular values as the diagonal elements are arranged in a non-decreasing manner, and H denotes the Hermitian transpose.

Step 3

Compute the predictor coefficient vector b:

$$b = (A)^{\#} h = \left(V \Sigma^{-1} U^H\right) h \tag{4}$$

where # denote the Moore–Penrose pseudo inversion.

Step 4

Define the polynomial $B(z)$:

$$B(z) = 1 - \sum_{i=1}^{L} b_i z^{-i}; M \leq L \leq N - M. \tag{5}$$

Step 5

The M roots $\{\mu_i, i = 1, 2, ..., M\}$ of $B(z)$ which fall on or outside the unit circle, out of the L roots $\{v_i, i = 1, 2, ..., L\}$, correspond to the reciprocal of the roots of $B(z)$:

$$\mu_i = 1/v_i; \ 1 \leq i \leq M.$$

Step 6

Compute the values of the signal parameters from the M roots of Step 5.

3. Proposed scheme

The implementation strategy of the proposed scheme is shown in Fig. 1. The parameters are estimated by the scheme in the following sequence. Initially, the backward linear

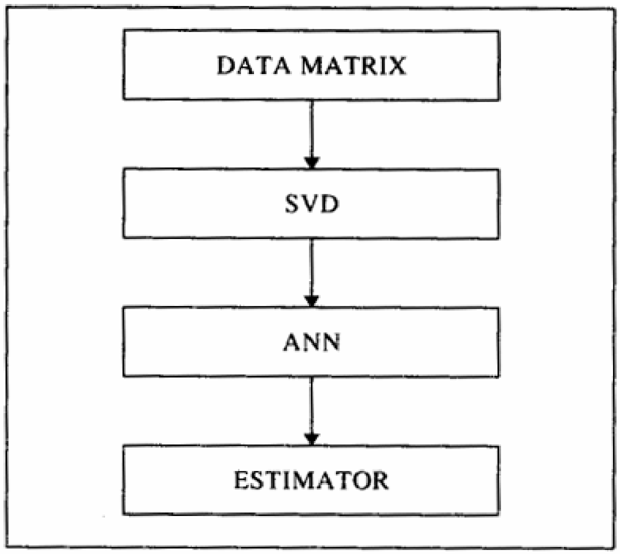

FIG. 1. Block diagram for ANN-based parameter estimator.

prediction (BLP) data matrix is formed from the signal samples. Then, the SVD algorithm breaks up the BLP data matrix into singular values and vectors, yielding biased singular values in the noisy environment. Now, these biased singular values are the inputs to the ANN block to produce the actual singular values, then the LPC algorithm described in Section 2 is used for the estimation of the parameters.

4. Artificial neural network (ANN) training

The neural network is trained before the scheme is exposed to the complex signal samples for parameter estimation. The training sets encompass the biased singular values as the inputs to the neural network, and the actual singular values which would have resulted without noise as the target. Although the singular vectors computed are affected at low SNRs of the signal, the estimated results predominantly depend on the singular values. For a particular signal with low noise power, the backward linear prediction data matrix is formed, and the conventional SVD along with LPC technique is employed for estimation of parameters such as amplitude, frequency, damping and phase. Singular values for the backward data matrix formed from the signal samples with varied noise power have been computed. The biased singular values so obtained from the noisy data along with the desired ones form the training pairs for learning the neural network. Training is terminated when the objective function, which is defined as half the sum of the square error, is minimum or below a certain floor level. The neural network has been exposed to different biased singular value sets corresponding to different corrupted signal samples for generalization. After completion of training the scheme is employed for estimation.

5. Simulation and result discussion

The number of hidden and output nodes of the network are judiciously selected with a view to reduce the computational burden. However, in this problem the number of nodes

Table I
Parameters of the ANN

Parameters	*Example 1* *Single exponentially damped sinusoids*	*Example 2* *Two exponentially damped sinusoids*
Number of data points, N	25	25
Filter order, L	4	8
Number of signals, M	2	2
Number of layers in the neural network	2	2
Number of inputs to the neural network	4	8
Number of output neurons	4	2
Number of hidden neurons	4	12

is selected based upon the filter order and the number of signals. It has been stated in Step 5 of the LPC algorithm in Section 2 that M out of the L roots corresponds to the signal parameters. So, the number of input to the network is chosen to be the same as the order of the filter and the number of output neurons is equal to the filter order or the number of input signals. Excellent results have been obtained with input samples corresponding to very low SNRs, which establish the robustness of the proposed scheme for a wide range of practical applications. Different types of signals with varying parameters such as damping, amplitude, frequency and phase are considered, and quite promising simulation results are obtained.

In this paper, two examples have been considered to verify the strength of the proposed method (Table I). The convergence characteristics for learning of the network of Example 1 has been illustrated in Fig. 2; the weights of the output and hidden layers are shown in Figs 3 and 4, respectively. At SNR of 10dB the estimated parameters are amplitude = 1; frequency = 0.45Hz; phase = 0.0; and damping = –0.0791.

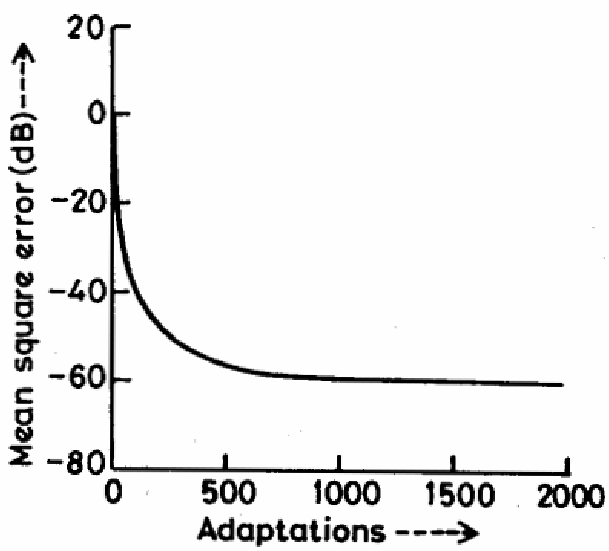

FIG. 2. Average learning curve for a damped sinusoidal signal.

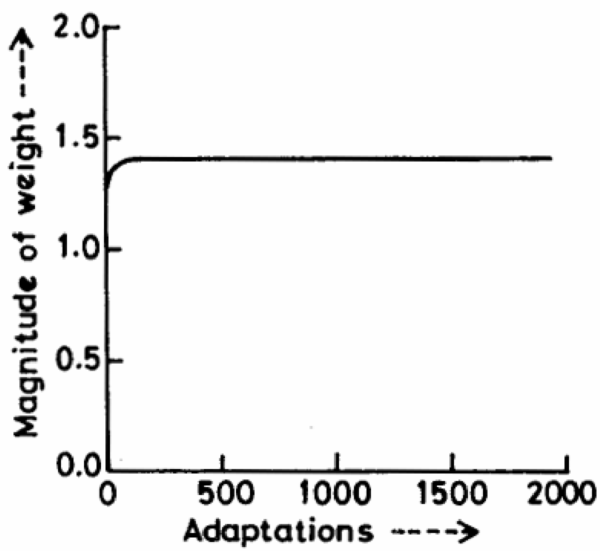

FIG. 3. Learning curve for output layer weight.

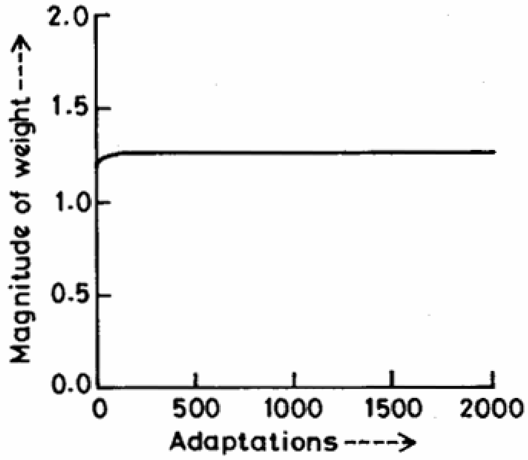

FIG. 4. Learning curve for hidden layer weight.

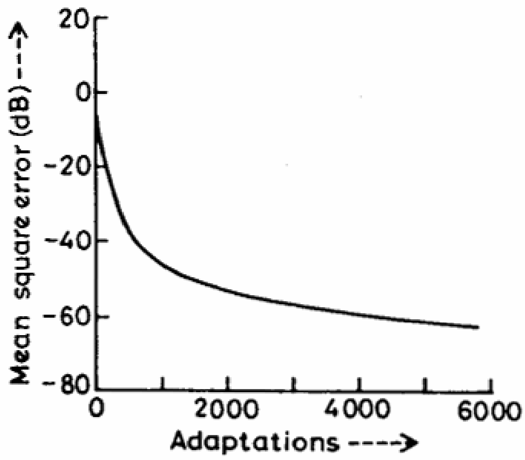

FIG. 5. Average learning curve for two damped sinusoidal signals.

The learning curve for the network in Example 2 has been illustrated in Fig. 5. For the above signals with an SNR of 10dB, the estimated values of the amplitudes are unity, frequencies are 0.45 and 0.35 Hz, and dampings are –0.1 and –0.2, respectively, with the associated phase angles 0.0.

5. Conclusions

ANNs along with LPC and SVD for parameter estimation of an exponentially damped sinusoidal signals have been presented in this paper. The results obtained exhibit the efficacy of the proposed method over the existing methods. It has been shown that the proposed modifications considerably reduce the problem of erroneous parameter estimation at very low SNRs.

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