KINEMATICS AND DYNAMICS OF CO-OPERATING MANIPULATORS ON A MOBILE BASE

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Abstract. This paper describes the kinematics and dynamics of multiple robotic arms mounted on a mobile base and co-operatively handling a common object. The presence of closed kinematic chains along with a mobile base of comparable mass and inertia with rest of the system makes the whole system dynamic analysis extremely complex. An attempt has been made to derive a complete unified dynamic model and the simulation algorithms for a general type of co-operative robotic system, with maximum emphasis on space robotic systems. All the derivations have been made in relation to space free-flying and free-floating robot manipulators.

1 Introduction and definition of notation

Co-operative robotic systems are an increasingly popular area of robotic systems research as they can perform tasks that cannot be easily handled by single manipulator. These tasks include handling large, heavy or non-rigid objects, assembly together of two or more separate components and space robotic applications. Analysis and control of co-ordinated systems is very complex due to the presence of inherent kinematic and dynamic interactions during execution of co-operative strategies.

In the case of fixed base co-operating manipulators, much work has been carried out [2, 4, 6]. However, many potential tasks in manufacturing and space robotics require the base to be mobile, and this makes the system even more complex. A mobile base with complete motion freedom, where the attitude can rotate about three axes as well as translate along spatial x, y and z axes, can be modelled as a six degree of freedom system in either free-flying or free-floating form. In the case of free-flying space robots, both the spacecraft and manipulator system are controlled simultaneously, whereas, once the spacecraft is positioned correctly, the spacecraft thruster system can be shut off to save fuel, and this is then known as a free-floating space robot.

The control of mobile-base robots is particularly challenging because of dynamic coupling when the mass and inertia of the base is comparable with that of the rest of the robotic system comprising the manipulators and the object handled. Past research on mobile-base systems has usually only involved a single manipulator, but some work has been reported recently on multiple manipulator systems [3], although this only covers the free-floating case and not the free-flying one.

This paper describes the derivation of a unified generalised dynamic model for a co-operating robotic system mounted on a mobile base of comparable mass and inertia, where the latter is subject to an external force. Unlike previous work, the model covers both the free-floating and free-flying cases. Simpler models can be readily derived from this generalised formulation. For example, for a fixed-base co-operating system, the base has zero mobility and infinite mass and inertia, and insertion of the appropriate base parameters produces a simplified model.

The general model of a co-ordinated robot manipulator system with m-robots, each with n_i links, installed on a moving base, is shown in Fig.1. The end-effectors hold a common object rigidly and it is assumed that the total mass and inertia of the robotic manipulators and the object is comparable with that of the mass and inertia of the base. To make the system more generalised, the model includes an external spatial force f_b applied to the base to represent the spacecraft thruster force that is met in free-flying space-robotics applications.

The co-ordinate frames are defined according to a modified form of the Denavit-Hartenberg convention such that the co-ordinate frame of a particular link is attached to that link with frame origin at the near end of the link. The spatial velocity, acceleration and force vectors of the ith link of the jth robot resolved in the ith link frame are denoted by (6×1) vectors ${}^{i}\mathbf{v}_j$, ${}^{i}\dot{\mathbf{v}}_j$ and ${}^{i}\mathbf{f}_j$. The 6 × 6 spatial transformation matrix ${}_{i-1}^{\;\;i}\mathbf{X}_j$ transforms a spatial vector from (i-1)th co-ordinate frame to the ith co-ordinate frame of the jth robot and is defined as [1]:

$$
{}_{i-1}^{\;\;i}\mathbf{X}_j = \begin{bmatrix} {}_{i-1}^{\;\;i}\mathbf{R}_j & \mathbf{0} \\ {}_{i-1}^{\;\;i}\mathbf{R}_j \, {}_{i-1}^{\;\;i}\tilde{\mathbf{p}}_j^T & {}_{i-1}^{\;\;i}\mathbf{R}_j \end{bmatrix} \quad \text{where:} \quad \tilde{\mathbf{p}} = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}
$$

and ${}_{i-1}^{\;i}\mathbf{R}_j$ is a 3 × 3 rotation matrix from the (i-1)th link frame to the ith link frame for the jth robot; ${}_{i-1}^{\;i}\mathbf{p}_j$ is a 3 × 1 vector from the origin of the (i-1)th link frame to the origin of the ith link frame for the jth robot.

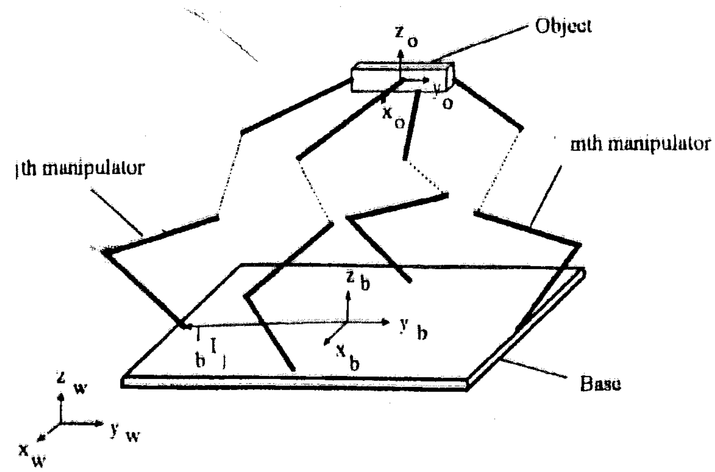

Fig 1: A co-operating robot system

The 6 × 6 spatial inertia matrix of the ith link of the jth robot is denoted by ${}^{i}\mathbf{M}_j$ and is defined as [1]:

$$ {}^{i}\mathbf{M}_j = \begin{bmatrix} -{}^{i}m_j\,{}_{i}^{c}\tilde{\mathbf{l}}_j & {}^{i}m_j\mathbf{E} \\ {}^{i}\mathbf{I}_j & {}^{i}m_j\,{}_{i}^{c}\tilde{\mathbf{l}}_j \end{bmatrix} $$

where ${}^{i}m_j$ is the mass of the ith link of the jth robot; ${}^{i}\mathbf{I}_j$ is the inertia tensor of the ith link of the jth robot at the ith frame origin; ${}_{i}^{c}\mathbf{l}_j$ is the distance from the ith frame origin to the centre of mass of the ith link of the jth robot; and $\mathbf{E}$ is an identity matrix.

2 Development of generalised model

Using orthogonal vectors ${}^{i}\Phi_j$ and ${}^{i}\Phi_j^{c}$ to represent the matrix of free and constrained mode vectors of the ith joint of the jth robot, the spatial velocity of the ith link of the jth robot represented in the ith link frame can be expressed in terms of the velocity of the base $\mathbf{V}_b$ as [5]:

$$ {}^{i}\mathbf{V}_j = {}_{b}^{i}\mathbf{X}_j\,\mathbf{V}_b + \sum_{k=1}^{i} {}_{k}^{i}\mathbf{X}_j\,{}^{k}\Phi_j\,{}^{k}\dot{q}_j + \sum_{k=1}^{i} {}_{k}^{i}\mathbf{X}_j\,{}^{k}\xi_j = {}_{b}^{i}\mathbf{X}_j\,\mathbf{V}_b + \sum_{k=1}^{i} {}_{k}^{i}\mathbf{X}_j\,({}^{k}\Phi_j\,{}^{k}\dot{q}_j + {}^{k}\xi_j) \quad (1) $$

The spatial velocity of all links of all m robots can be expressed as: $\mathbf{V} = \mathbf{X}_b\,\mathbf{V}_b + \mathbf{X}(\Phi\,\dot{q} + \xi)$ (2)

where $\mathbf{V} = [\mathbf{V}_1^T\ \mathbf{V}_2^T \ldots \mathbf{V}_m^T]^T$, $\mathbf{X}_b = [{}_b\mathbf{X}_1^T\ {}_b\mathbf{X}_2^T \ldots {}_b\mathbf{X}_m^T]^T$, $\mathbf{X} = \mathrm{diag}(\mathbf{X}_1\ \mathbf{X}_2 \ldots \mathbf{X}_m)$, $\Phi = \mathrm{diag}(\Phi_1\ \Phi_2 \ldots \Phi_m)$, $\dot{q} = [\dot{q}_1^T\ \dot{q}_2^T \ldots \dot{q}_m^T]^T$, and $\xi = [\xi_1^T\ \xi_2^T \ldots \xi_m^T]^T$.

The end-effector velocity of the jth robot, denoted by v_j^e, can be expressed in the base frame as:

$$ v_j^e = {}_b\mathbf{R}_j\,{}_{b}^{n+1}\mathbf{X}_j\,\mathbf{V}_b + {}_b\mathbf{R}_j\,{}_{n}^{n+1}\mathbf{X}_j\,\mathbf{L}_j(\Phi_j\,\dot{q}_j + \xi_j) = \mathbf{J}_{bj}\,\mathbf{V}_b + \mathbf{J}_{qj}\,\dot{q}_j + \mathbf{k}_j\,\xi_j \quad (3) $$

where ${}_b\mathbf{R}_j = \mathrm{diag}({}_{n+1}^{\;b}\mathbf{R}_j\ {}_{n+1}^{\;b}\mathbf{R}_j)$ with ${}_{n+1}^{\;b}\mathbf{R}_j$ is a rotation matrix from the end-effector frame to the base frame. $\mathbf{k}_j = {}_b\mathbf{R}_j\,{}_{n}^{n+1}\mathbf{X}_j\,\mathbf{L}_j$, $\mathbf{J}_{qj} = {}_b\mathbf{R}_j\,{}_{n}^{n+1}\mathbf{X}_j\,\mathbf{L}_j\,\Phi_j$, and $\mathbf{J}_{bj} = {}_b\mathbf{R}_j\,{}_{b}^{n+1}\mathbf{X}_j$.

For all the m-robots, Eq.(3) can be represented as: $v^e = \mathbf{J}_b\,\mathbf{V}_b + \mathbf{J}_q\,\dot{q} + \mathbf{k}\,\xi$ (4)

The momentum of the entire system with respect to the inertial reference frame can be expressed as

$$ \mathbf{P} = ({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}_b' + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)\mathbf{V}_b + (\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\Phi + {}_w\mathbf{M}_o\,\mathbf{J}_q')\dot{q} + (\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o\,\mathbf{k}')\xi \quad (5) $$

For a free-floating space robotic system, the total momentum of the system P will be zero, therefore from Eq.(5) the base velocity can be expressed as:

$$\mathbf{V}_b = -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}\{(\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\Phi + {}_w\mathbf{M}_o\,\mathbf{J}'_q)\dot{\mathbf{q}} + (\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o\,\mathbf{k}')\xi\}$$
$$= \mathbf{J}_f\,\dot{\mathbf{q}} + \mathbf{k}_1\,\xi \tag{6}$$

where $\mathbf{J}_f = -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}(\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\Phi + {}_w\mathbf{M}_o\,\mathbf{J}'_q)$ and

$\mathbf{k}_1 = -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}(\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o\,\mathbf{k}')$.

The m end-effector velocity vector of a free-floating robotic system can be derived using Eqs.(4) and (6):

$$\mathbf{v}^* = (\mathbf{J}_b\,\mathbf{J}_f + \mathbf{J}_q)\dot{\mathbf{q}} + (\mathbf{J}_b\,\mathbf{k}_1 + \mathbf{k})\xi = \mathbf{J}\,\dot{\mathbf{q}} + \mathbf{k}_o\,\xi \tag{7}$$

For free-flying robotic systems, the above relationship is not valid since momentum is not conserved, and the external force $\mathbf{f}_b$ becomes very important. This can be expressed as the rate of change of total momentum $\mathbf{P}$:

$$\mathbf{f}_b = \dot{\mathbf{P}} = ({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)\dot{\mathbf{V}}_b + (\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\Phi + {}_w\mathbf{M}_o\,\mathbf{J}'_q)\ddot{\mathbf{q}} + \mathbf{b}_f \tag{8}$$

where $\mathbf{b}_f$ is the net bias force acting on the system, which is a function of position, velocity and time.

The acceleration of the manipulators links can be obtained by differentiating Eq.(2) to give:

$$\dot{\mathbf{V}} = \mathbf{X}\,\Phi\,\ddot{\mathbf{q}} + \dot{\mathbf{X}}_b\,\mathbf{V}_b + \mathbf{X}_b\,\dot{\mathbf{V}}_b + \zeta \quad \text{where } \zeta = \dot{\mathbf{X}}(\Phi\,\dot{\mathbf{q}} + \xi) + \mathbf{X}(\dot{\Phi}\,\dot{\mathbf{q}} + \dot{\xi}).$$

The force exerted on the ith link of the jth robot is expressed as:

$$^{i}\mathbf{f}_j = {}_{n+1}^{\;\;i}\mathbf{X}_j\,^{n+1}\mathbf{f}_j + \sum_{k=i}^{n} {}_k^i\mathbf{X}_j\,(^k\mathbf{M}_j\,^k\dot{\mathbf{V}}_j + {}^k\mathbf{b}_j) \tag{9}$$

Now the force vector for the jth robot can be described as: $\mathbf{f}_j = \mathbf{X}_j^T(\mathbf{D}_j\,^{n+1}\mathbf{f}_j + \mathbf{M}_j\,\dot{\mathbf{V}}_j + \mathbf{b}_j)$, where $\mathbf{f}_j = [^1\mathbf{f}_j^T\,^2\mathbf{f}_j^T \ldots ^n\mathbf{f}_j^T]^T$, $\dot{\mathbf{V}}_j = [^1\dot{\mathbf{V}}_j^T\,^2\dot{\mathbf{V}}_j^T \ldots ^n\dot{\mathbf{V}}_j^T]^T$, $\mathbf{b}_j = [^1\mathbf{b}_j^T\,^2\mathbf{b}_j^T \ldots ^n\mathbf{b}_j^T]^T$, $\mathbf{D}_j = [0 \ldots 0\ {}^{n+1}_{\;\;n}\mathbf{X}_j]^T$ and $\mathbf{M}_j = \mathrm{diag}(^1\mathbf{M}_j\,^2\mathbf{M}_j \ldots ^n\mathbf{M}_j)$. For all the m robots this force relationship can be expressed as:

$$\mathbf{f} - \mathbf{X}^T\,\mathbf{D}\,\mathbf{f}_e = \mathbf{X}^T(\mathbf{M}_q\,\dot{\mathbf{V}} + \mathbf{b}) \tag{10}$$

where $\mathbf{f} = [\mathbf{f}_1^T\,\mathbf{f}_2^T \ldots \mathbf{f}_m^T]^T$, $\dot{\mathbf{V}} = [\dot{\mathbf{V}}_1^T\,\dot{\mathbf{V}}_2^T \ldots \dot{\mathbf{V}}_m^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T\,\mathbf{b}_2^T \ldots \mathbf{b}_m^T]^T$, $\mathbf{D} = \mathrm{diag}(\mathbf{D}_1\,\mathbf{D}_2 \ldots \mathbf{D}_m)$, $\mathbf{M}_q = \mathrm{diag}(\mathbf{M}_1\,\mathbf{M}_2 \ldots \mathbf{M}_m)$, and $\mathbf{f}_e$ is the force exerted by the end-effector on the object, which can be expressed by the relationship $\mathbf{f}_e = [^{n+1}\mathbf{f}_1^T\,^{n+1}\mathbf{f}_2^T \ldots ^{n+1}\mathbf{f}_m^T]^T$.

From Eq.(9) the force exerted by the base on the first link of the jth robot can be expressed as:

$$^{1}\mathbf{f}_j = {}_{n+1}^{\;\;1}\mathbf{X}_j\,^{n+1}\mathbf{f}_j + \sum_{k=1}^{n} {}_k^1\mathbf{X}_j\,(^k\mathbf{M}_j\,^k\dot{\mathbf{V}}_j + {}^k\mathbf{b}_j) \tag{11}$$

The force equilibrium equation of the base with an external force acting on it, can be represented as

$$\mathbf{f}_b = \mathbf{M}_b\,\dot{\mathbf{V}}_b + \mathbf{b}_b + \mathbf{X}_b^T(\mathbf{D}\,\mathbf{f}_e + \mathbf{M}_q\,\dot{\mathbf{V}} + \mathbf{b}) \tag{12}$$

Now solving Eq.(12) for the base acceleration and substituting the value of $\dot{\mathbf{V}}_b$, the acceleration of the manipulators can be expressed as

$$\dot{\mathbf{V}} = (\mathbf{E} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{M}_q)^{-1}\{\mathbf{X}\,\Phi\,\ddot{\mathbf{q}} + \dot{\mathbf{X}}_b\,\mathbf{V}_b - \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T(\mathbf{D}\,\mathbf{f}_e + \mathbf{b}) + \mathbf{X}_b\,\mathbf{M}_b^{-1}(\mathbf{f}_b - \mathbf{b}_b) + \zeta\} \tag{13}$$

Eliminating $\dot{\mathbf{V}}$ from Eqs.(10) and (13) gives:

$$\mathbf{f} - \mathbf{X}^T(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1}\mathbf{M}_q^{-1}\,\mathbf{D}\,\mathbf{f}_e = \mathbf{X}^T(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1}\{\mathbf{X}\,\Phi\,\ddot{\mathbf{q}} + \dot{\mathbf{X}}_b\,\mathbf{V}_b + \mathbf{M}_q^{-1}\,\mathbf{b} + \mathbf{X}_b\,\mathbf{M}_b^{-1}(\mathbf{f}_b - \mathbf{b}_b) + \zeta\} \tag{14}$$

where $(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1} = \mathbf{M}_q(\mathbf{E} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{M}_q)^{-1}$.

Multiplying both the sides of the Eq.(14) with Φ^T and rearranging leads to a concise representation of the joint torque vector as:

$$\mathbf{T} - \mathbf{J}^T\,\mathbf{f}_e = \mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}. \tag{15}$$

where $\mathbf{M} = \Phi^T\,\mathbf{X}^T(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1}\mathbf{X}\,\Phi$, $\mathbf{J}^T = \Phi^T\,\mathbf{X}^T(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1}\mathbf{M}_q^{-1}\,\mathbf{D}$,

$\mathbf{C} = \Phi^T\,\mathbf{X}^T(\mathbf{M}_q^{-1} + \mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1}\{\dot{\mathbf{X}}_b\,\mathbf{V}_b + \mathbf{M}_q^{-1}\,\mathbf{b} + \mathbf{X}_b\,\mathbf{M}_b^{-1}(\mathbf{f}_b - \mathbf{b}_b) + \zeta\}$, and $\mathbf{T} = \Phi^T\,\mathbf{f}$.

The inversion of the matrix in the Eq.(14) can be simplified using a matrix inversion lemma:

$$(\mathbf{M}_q^{-1}+\mathbf{X}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T)^{-1} = \mathbf{M}_q - \mathbf{M}_q\,\mathbf{X}_b(\mathbf{M}_b + \mathbf{X}_b^T\,\mathbf{M}_b\,\mathbf{X}_b)^{-1}\mathbf{X}_b^T\,\mathbf{M}_q$$

If the object is assumed to be held rigidly by m manipulators, then the force at the centre of mass of the object due to all the end-effector forces acting on it, can be represented as [7]: $\mathbf{f}_o = \mathbf{W}^T\,\mathbf{f}_e$ (16)

The force balance equation for this object can be represented as: $\mathbf{f}_o = \mathbf{M}_o\,\dot{\mathbf{v}}^o + \mathbf{b}_o$ (17)

where $\mathbf{b}_o$ is the bias force on the object. Combining Eqs.(16) and (17) now leads to the following dynamic equation for the object: $\mathbf{M}_o\,\dot{\mathbf{v}}^o + \mathbf{b}_o = \mathbf{W}^T\,\mathbf{f}_e$ (18)

The forward dynamics analysis of the co-operating manipulators on a mobile base can be described with reference to Eq.(15), which involves the computation of the joint accelerations $\ddot{\mathbf{q}}$ with the knowledge of the input torques and forces, $\mathbf{T}$, current state of the manipulator, $\mathbf{q}$, $\dot{\mathbf{q}}$ and motion of the base. Simplifying Eq.(8) for $\dot{\mathbf{V}}_b$ and carrying out further simplification leads to the following expression for the end effector acceleration $\dot{\mathbf{v}}^e$:

$$\begin{aligned}\dot{\mathbf{v}}^e = &\{\mathbf{J}_q - \mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{M}_q(\mathbf{X}\,\Phi - \mathbf{X}_b\,\mathbf{GH})\}\ddot{\mathbf{q}} + (\dot{\mathbf{J}}_b - \mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{M}_q\,\dot{\mathbf{X}}_b)\mathbf{V}_b\\ &+ \mathbf{J}_b\,\mathbf{M}_b^{-1}(\mathbf{E} - \mathbf{X}_b^T\,\mathbf{M}_q\,\mathbf{G})\,\mathbf{f}_b + \mathbf{J}_b\,\mathbf{M}_b^{-1}(\mathbf{X}_b^T\,\mathbf{M}_q\,\mathbf{G}\,\mathbf{b}_r - \mathbf{b}_b - \mathbf{X}_b^T\,\mathbf{b} + \zeta)\\ &+ (\dot{\mathbf{J}}_q\,\dot{\mathbf{q}} + \mathbf{k}\,\dot{\xi} + \dot{\mathbf{k}}\,\xi) - \mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{D}\,\mathbf{f}_e\\ = &\;\dot{\mathbf{v}}^e_{open} - \dot{\mathbf{v}}^e_{constrained}\end{aligned} \quad (19)$$

Hence, the system can be modelled as a superposition of open chain part and a constrained part due to the presence of Cupertino. The following explicit relationship between the end-effector force and object acceleration can now be obtained:

$$\dot{\mathbf{v}}^e_{constrained} = \dot{\mathbf{v}}^e_{open} - \mathbf{W}\,\dot{\mathbf{v}}^o - \dot{\mathbf{W}}\,\mathbf{v}^o \quad \Rightarrow \mathbf{f}_e = (\mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{D})^{-1}[\dot{\mathbf{v}}^e_{open} - \mathbf{W}\,\dot{\mathbf{v}}^o - \dot{\mathbf{W}}\,\mathbf{v}^o] \quad (20)$$

Then, substituting the value of $\mathbf{f}_e$ from Eq.(20) into Eq.(18) gives:

$$\dot{\mathbf{v}}^o = [\mathbf{M}_o + \mathbf{W}^T(\mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{D})^{-1}\mathbf{W}]^{-1}\{\mathbf{W}^T(\mathbf{J}_b\,\mathbf{M}_b^{-1}\,\mathbf{X}_b^T\,\mathbf{D})^{-1}\;(\dot{\mathbf{v}}^e_{open} - \dot{\mathbf{W}}\,\mathbf{v}^o) - \mathbf{b}_o\} \quad (21)$$

Once the spatial acceleration of the object $\dot{\mathbf{v}}^o$ is known, Eq.(20) can give all the end-effector spatial forces. Similarly from Eq.(15), $\ddot{\mathbf{q}}$ can be represented as: $\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{T} - \mathbf{C}) - \mathbf{J}^T\,\mathbf{f}_e = \ddot{\mathbf{q}}_{open} - \ddot{\mathbf{q}}_{constrained}$ (22)

Hence, the joint accelerations of the entire system can be computed with the known open chain joint accelerations and end effector forces.

3 Conclusions

In this paper, a unified approach for the kinematics and dynamics of a co-operating robotic system on a mobile base has been presented, with special emphasis on space robotics. In the presence of closed kinematic chain constraints, the kinematics and dynamics of space robots become increasingly complex. Both inverse dynamics and forward dynamics of these systems have been addressed. In addition, it has been shown that the simulation of this type of system can be carried out using any efficient multi-arm unconstrained analysis approaches.

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