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Parameter estimation of power system signals using SVD

This paper presents an estimation technique for amplitude, frequency, phase and damping of a signal embedded in noise. This signal considered in this paper belongs to that of a power system, where frequency deviations in the neighbourhood of the nominal value occur due to dynamic oscillations or faults. The off-nominal frequency, amplitude and damping are estimated using a damped sinusoidal model, and a Taylor series expansion technique around the nominal value. Singular value decomposition (SVD) technique is used to solve the set of over-determined and under-determined equations for the instantaneous data samples, and the relevant computer simulation results are presented.

Introduction

The estimation of the signal parameters including the frequency, amplitude, phase and the degree of damping can be broadly classified into parametric and non-parametric methods. The non-parametric methods make no assumptions how the data were generated, and the estimates are based entirely on a finite record of data. The data is assumed to be either zero outside the measurement interval, or to be periodic. A finite record causes spectral leakage and poor frequency resolution, which can be alleviated by increasing the record length. However, a large record length may not be suitable for on-line applications, especially when the signal is non-stationary or transient. The parametric method on the other hand assumes a model for the signal generation, and the parameters of the model are estimated from the observed data. The parametric methods give significantly higher resolution, and do not suffer from leakage effects due to frequency sampling (Doraiswami and Liu, 1990; Proakis and Manolakis, 1988).

In power systems, it is essential to maintain the frequency of the system close to its nominal value, usually, frequency deviations in the range of two or three per cent are only allowed for short durations of time. In this range, the least square error algorithm provides accurate estimate of the amplitude, frequency and phase of the signal, and thus, is the most suitable for off-nominal frequency estimation and relaying using a microcomputer. Other methods to estimate the frequency and degree of damping of signal are based on Fourier Transformation (Lee and Poon, 1989), are extended prony's method (Wang et al., 1988; Hauer, 1988). The eigen value method is a parametric method and requires the knowledge of the system matrix representing the state variable model of the power system, and hence,it is unsuitable for on-line applications. The Fourier Transform which is a non-parametric method, although suitable for on-line applications, requires sufficiently large data record compared to the period of the lowest frequency components, so that the leakage effect is negligible and the frequency resolution high.

In this paper, the off-nominal frequency and amplitude of a power system signal like the voltage or current is estimated using a wave form modelling technique. The parameters of a modelled wave form (assuming it to be a damped sinusoid) are estimated from the Wiener-Hopf equation which is solved using singular value decomposition (SVD) (Doraiswami and Jiang, 1989; Swain, 1991; Klema et al., 1980) technique. SVD yields a numerically robust solution, and is more accurate in comparison to the least square algorithm. The efficacy of the technique is tested by using computer simulation for a power system signal, whose frequency varies between 40 to 60 Hz range. Further the paper explores the effect of sampling rate, magnitude of damping, etc. by corrupting the signal with a white noise of varying noise power.

Signal model

The general power system signal can be described as a damped sinusoid, which can be modelled as

$$y(t) = A\,e^{-\sigma t} \sin(2\pi f t + \varnothing) \quad \ldots \quad (1)$$

where y(t) is the intantaneous value of the signal, A is the peak value of the signal, f is the frequency of the signal which may be either voltage or current, $\varnothing$ is the arbitrary phase angle, and σ is the damping factor.

Equation (1) can be expanded in the neighbourhood of the nominal frequency F_o, by using Taylor series expansion, which gives

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$$y(t) = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + \ldots + a_{18}x_{18} \quad \ldots \quad (2)$$

where

$$\left.\begin{array}{l}
x_1 = A\cos\varnothing,\ \ x_2 = (\Delta f)\ A\cos\varnothing,\\
x_3 = A\sin\varnothing,\ \ x_4 = (\Delta f)\ A\sin\varnothing,\\
x_5 = (\Delta f)^2\ A\cos\varnothing,\ \ x_6 = (\Delta\varnothing)^2\ A\sin\varnothing,\\
x_7 = -\sigma\ A\cos\varnothing,\ \ x_8 = -\sigma\ \Delta f\cos\varnothing,\\
x_9 = -\sigma\ A\sin\varnothing,\ \ x_{10} = -\sigma\ (\Delta f)\ A\sin\varnothing,\\
x_{11} = -\sigma\ (\Delta f)^2\ A\cos\varnothing,\\
x_{12} = -\sigma\ (\Delta f)^2\ A\cos\varnothing,\\
x_{13} = -(\sigma^2/2.0)\ A\cos\varnothing,\\
x_{14} = -(\sigma^2/2.0)\ \Delta f\ A\cos\varnothing,\\
x_{15} = -(\sigma^2/2.0)\ A\sin\varnothing,\\
x_{16} = -(\sigma^2/2.0)\ \Delta f\ A\sin\varnothing,\\
x_{17} = -(\sigma^2/2.0)\ (\Delta f)^2\ A\cos\varnothing,
\end{array}\right\} \quad (3)$$

$$x_{18} = -(\sigma^2/2.0)\ (\Delta f)^2\ A\cos\varnothing,$$

$$\left.\begin{array}{l}
a_1 = \sin(2\pi f_o t),\ \ a_2 = 2\pi t\cos(2\pi f_o t),\\
a_3 = \cos(2\pi f_o t),\ \ a_4 = 2\pi t\sin(2\pi f_o t),\\
a_5 = -2(\pi t)^2\sin(2\pi f_o t),\ \ a_6 = -2(\pi t)^2\cos(2\pi f_o t),\\
a_7 = t\sin(2\pi f_o t),\ \ a_8 = 2\pi t^2\cos(2\pi f_o t),\\
a_9 = t\cos(2\pi f_o t),\ \ a_{10} = -2\pi t^2\sin(2\pi f_o t),\\
a_{11} = -2\pi^2 t^3\sin(2\pi f_o t),\ \ a_{12} = -2\pi^2 t^3\cos(2\pi f_o t),\\
a_{13} = t^2\sin(2\pi f_o t),\ \ a_{14} = -2\pi t^3\cos(2\pi f_o t),\\
a_{15} = t^2\cos(2\pi f_o t),\ \ a_{16} = -2\pi t^3\sin(2\pi f_o t),\\
a_{17} = -2\pi^2 t^4\sin(2\pi f_o t),\ \ a_{18} = -2\pi^2 t^4\sin(2\pi f_o t),
\end{array}\right\} \quad (4)$$

$$\Delta f = f - f_o \quad (5)$$

If the signal is sampled at t_1, $t_1 + \Delta t$ and $t_1 + 2\Delta t$ seconds etc., the left hand side of equation (3) gives the signal values at the above intervals of time. Since time is an arbitrary quantity, and t_1 can be assigned a value, the 'a' coefficients of equation (2) are known and the 'x' components are unknown. Both damped and undamped signals are considered in this paper. If damping is small, $e^{-\sigma t} \cong 1.0$, and hence only six equations will be adequate to determine the unknown values of x's. However, it is well known that an over-determined set of equations is required to determine the signal magnitude and frequency, as the signal model is corrupted by white noise of variance, τ and SNR of ρ Equation (6) expresses m such equations in n unknown in the matrix form as

$$[A]\ [x] = [y] \quad \ldots.\ (6)$$

where [y] is a set of vector measurements of the instantaneous values of the signal, [x] is the vector of unknown from which amplitude, frequency, phase and damping of the signal are estimated, and [A] is the coefficient matrix whose elements are known. For m>n or m<n, the vector x is obtained by minimising the least squares equation (Cadzow, 1984 and 1990)

$$J(w) = (Y - AX)^T\ (Y - AX) \quad \ldots\ (7)$$

where Y is the data matrix $[y(0)y(1)\ \ldots..y(m)]^T$.

The optimal tap weight vector is given by

$$A^T Ax = A^T Y \quad \ldots\ (8)$$

The solution of the Wiener-Hopf equation is obtained using SVD. SVD yields a numerically robust solution, and its use is necessary since the matrix A is not likely to have a full rank (Klema et al., 1980, Ezio and Kung, 1989; Strang, 1980). Therefore,

$$A = U\begin{bmatrix}\Sigma & 0\\ 0 & 0\end{bmatrix}V^T \quad \ldots\ (9)$$

where U and V are of dimensions (m x m) and (n x n), respectively, and of the form

$$U = [u_1, u_2, \ldots\ldots\ldots, u_m],\ \text{and}\ V = [v_1, v_2, \ldots\ldots\ldots, v_n],$$

and Σ is an rxr matrix of the form (diag. $[\lambda_1, \lambda_2, \ldots, \lambda_r]$) such that $\lambda_1 \geq \lambda_2, \ldots \geq \lambda_r$, and $\lambda_{r+1} = \ldots\ldots\ldots = \lambda_1 = 0$, and $r \leq 1 = \min(m, n)$ is the rank of the matrix A.

This culminates in an expression for the estimated weights which minimizes the least squared error

$$x = -\sum_{k=1}^{r} \lambda_k v_k u_k^T Y \quad \ldots\ (10)$$

The factors that affect the suitability of technique are the size of the data window, the sampling frequency, and the truncation of the Taylor series expressions of the sine and cosine terms. A considerable freedom in selecting these parameters is considered in this paper to estimate the amplitude, phase and frequency of a signal embedded in noise, and the type of the signal considered belongs to power system.

Once the x values are obtained by multiplying the weighting factors with a sliding data window, the amplitude, frequency and damping (Giray and Sachdev, 1989; Sachdev and Giray, 1980) of the signal are obtained as

$$A = \sqrt{x_1^2 + x_3^2} \quad \ldots\ (11)$$

$$\Delta f = x_2/x_1 = x_4/x_3 = \sqrt{x_5/x_1} = \sqrt{x_6/x_3}$$

$$\Delta f = \sqrt{(x_5^2 + x_6^2)/(x_2^2 + x_4^2)} \quad \ldots\ (12)$$

$$\sigma = -\frac{x_7}{x_1} = -\frac{x_8}{x_2} = \sqrt{(x_7^2 + x_9^2)/(x_1^2 + x_3^2)} \quad \ldots.\ (13)$$

and so on.

Several such formulae can be desired for determining the above parameters f, and σ. Out of these the first two formulations for frequency deviation calculation is accurate because the higher order terms amplify noise more than that of the lower orders. In this paper, the frequency deviation is determined by averaging the first two formulations, and it is observed that this results in accurate estimation for a wide range of frequencies.

As the data samples are discrete, and the above combinations are nothing but the use of non-recursive digital filters, certain error creep into the estimates of the above quantities. The errors are much pronounced in a least square error technique, where the pseudo-inverse of the matrix A is used to solve the over-determined equation. However, with SVD the solution seems to be robust and more accurate.

The least mean square and average error criteria used for prediction of performance is as follows

$$J = (1/N)\sqrt{\sum_{k=0}^{N} e^{2}(k)} \quad \text{... (14)}$$

and

$$J_{mean} = (1/N)\sum_{k=0}^{N} e(k) \quad \text{... (15)}$$

These two criteria are used to predict the value of the frequency and the signal amplitude at a future time, and are thus suitable for building intelligent sensors and an expert system for heuristic assessment of signal parameters.

Computer simulation

Computer simulation results are obtained from the simulated data of a damped voltage wave form corrupted with white noise. The damping magnitude and signal to noise ratio are varied. The frequency of the voltage signal is taken as 50 Hz, and the nominal frequency is varied from 40 Hz to 60 Hz and vice-versa. The sampling rate is varied from 400 Hz to 1600 Hz and filter order is either 6 or 18. At first a minimum data point of 6 is used for evaluating the amplitude and frequency (damping is neglected). However, in the final simulation the data points are varied to 38, and the signal parameters are estimated. The signal to noise ratio is varied from 40 dB upto 0 dB. At 30 dB and 20 data points the frequency estimate is excellent. At low SNR, together with prior knowledge of system frequency, the actual frequency is estimated by suitable initial nominal frequency.

Results and discussion

Fig. 2 compares the effect of sampling frequencies on the amplitude estimates of the signal. It is seen from this figure that as the sampling frequency increases from 400 Hz to 800 Hz, the estimated amplitude becomes more accurate in the frequency range of 40 Hz to 55 Hz, i.e., for a deviation of +10 Hz from actual frequency values. However, if the frequency becomes equal to 1600 Hz, the amplitude versus frequency response is found to be oscillatory. Also the estimation of signal frequency is found to be better at 800 Hz (16 samples per cycle) than either at 400 or 1600 Hz.

At higher value of data samples used for weight calculations, the amplitude estimate improves dramatically. However, the frequency error with different sampling frequency and filter order is shown in Fig. 1. It is observed that at lesser number of data points (i.e., 6) and with sampling frequency fixed at 400 Hz, the frequency estimate is found to be accurate from 45 to 55 Hz. Whereas, when the sampling frequency increases to 800 Hz, the accuracy deteriorates to the 47.5 Hz to 52.5 Hz, i.e., the range of frequency deviation of 5 Hz. With more data samples and increased sampling frequency the frequency estimate is found to be better, but with high sampling frequency and less number of data samples (near to the filter order) the frequency estimate is found to be proper.

Further increasing the noise amplitudes, it is found that more number of data samples are required for an accurate estimate of signal amplitude and frequency. The same concept is used to evaluate damping under high noise condition, i.e., low SNR.

Conclusion

The paper presents a simple approach for the estimate of amplitude, frequency, phase and damping of a signal embedded in noise. The frequency deviation from nominal value usually occurs in a power system when faults and sudden load changes cause dynamic oscillations of the system. The paper highlights the effects of sampling frequency, data length, damping magnitude and SNR on the band width of the accurate frequency estimation near the nominal value. The SVD technique is found to be robust and yields an accurate estimate of the signal parameters of the power system signal. This paper presents a generalised signal processing technique which can be applied to other signals as well. Further, a comparison with linear predictive coding (LPC) approach of identifying a signal being worked out in a future paper. This technique is found to be a simple one and is being applied for microcomputer based frequency monitoring in power and communication systems. However, further research work can be carried out to overcome the limitations of the proposed scheme for low SNR and high damping coefficients.

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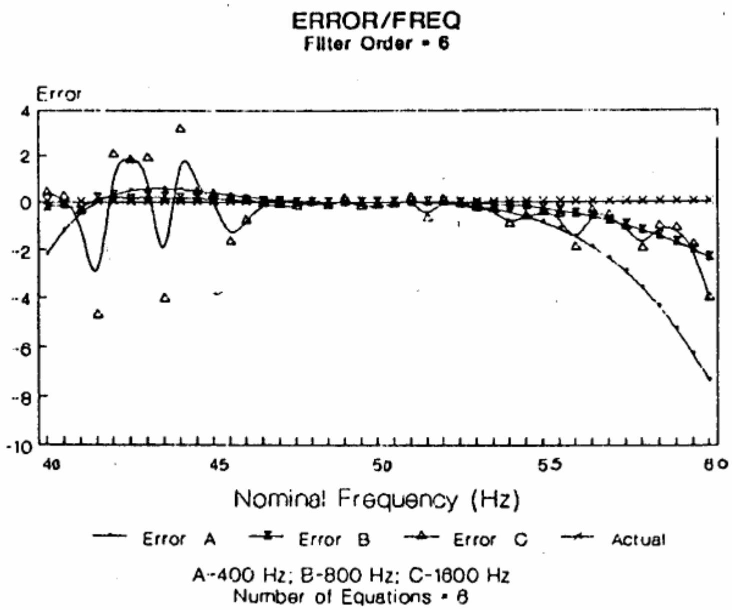

Fig.1 Error in frequency estimate

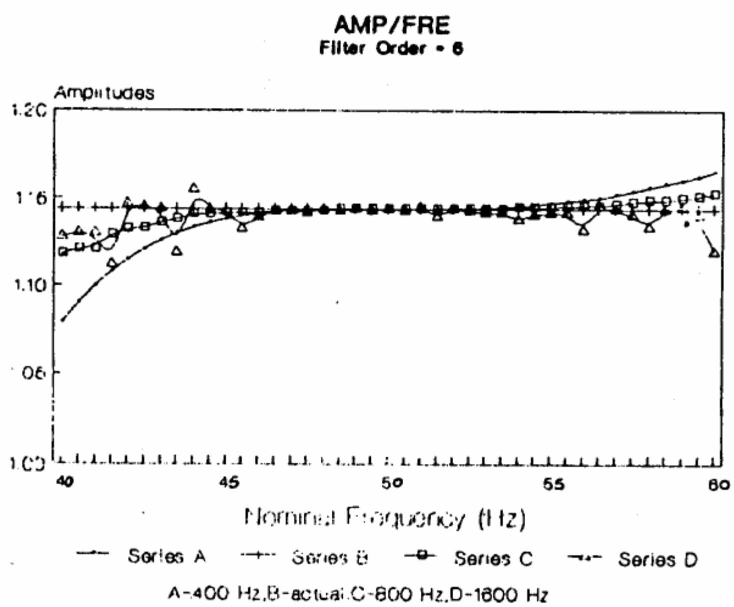

Fig.2 Amplitude estimation

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