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A special class of distorted premium principle based on an extension of the exponential-geometric distribution

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Abstract

In this paper a new probability density function with both unbounded and bounded support is presented. The new distribution, called modified exponential-geometric distribution arises from the exponential-geomeric distribution introduced by Adamidis and Loukas [1]. It presents a variety of shapes of density function and hazard rate function. The distribution with scale-transformed bounded support is considered as an alternative to the classical beta distribution and is shown to have an application in insurance. In particular, we suggest a special class of distorted premium principle based on this distribution and we compare it with the dual power premium principle. Moreover, the proposed distribution with unbounded support is used as a lifetime model and is considered as an attractive alternative to some existing models in the reliability literature.

Keywords and Phrases Distortion function; Hazard rate function; Maximum likelihood estimation; Monte-Carlo simulation

1 Introduction

In insurance, a probability distribution with domain on $(0, 1)$ can be used as a distortion function to define a premium principle. This is why the classical beta distribution has a dominant role in insurance to produce a class of beta-distorted premium principles. For detail see

Section 2.6 in Denuit et al. [4]. Although many researchers have proposed probability distributions with domain on (0,1), most of these distributions involved special functions in their formulations except Kumaraswamy distribution (see Jones [9]) and hence was not probably considered as a distortion function. A review of the distributions with domain on (0,1) can be found in Nadarajah [13]. Recently, Gómez et al. [7] use the log-Lindley distribution as an alternative to the beta distribution to produce a class of distorted premium principle.

The exponential-geomeric (EG) distribution with unbounded support has been presented by Adamidis and Loukas [1] as a lifetime model with decreasing failure rate. It is one of the earliest published papers on lifetime distributions of a system with random number of components. Here the authors assumed lifetime of individual components to follow *iid* exponential distribution, and model the number of components failed by zero-truncated geometric distribution. While the EG distribution has been proven to be quite effective to model any lifetime behaviour with decreasing hazard function, it fails to model lifetime with other forms of hazard rates. Encouraged by the findings of Gómez et al. [7] in the context of distortion premium principle, we take the work of Adamidis and Loukas [1] a step further in a new direction and introduce a new probability distribution with both unbounded and bounded support. While the new distribution with bounded support on (0,1) is used as a distortion function, the same with unbounded support has competitive features for lifetime modeling. This distribution, namely modified exponential-geomeric (MEG) distribution is derived from the EG distribution as mentioned earlier with the probability density function (PDF) given by

$$f_{EG}(x;\beta,p) = \beta(1-p)e^{-\beta x}(1-pe^{-\beta x})^{-2};\ x>0,\ \beta>0,\ 0<p<1.$$

The proposed MEG distribution exhibits varying shapes of density function including U shape, with increasing, decreasing and bath-tub failure rates. The bounded support of the MEG distribution is further transformed into $(0,1)$ support to have another distribution namely beta-equivalent modified exponential-geomeric (MEG_B) distribution which is shown to have application in insurance.

The rest of the paper is organized as follows. In Section 2, the MEG distribution is derived from the EG distribution. The shapes and the hazard rates of the distribution are derived and discussed in detail along with moments, percentiles and coefficients of variation (CV). Parameters of the distribution are estimated by the maximum likelihood method through a simulation study. A real life example from reliability is provided with a detailed comparison with some other competitive distributions. In Section 3, the MEG_b distribution is introduced with scale-transformed bounded support $(0,1)$. One application in insurance is shown in detail. It is shown that the MEG_B distribution induces a principle whose premium exceeds the net premium (or expected risk) and sometimes is, for appropriate choice of the parameters, less than the dual power premium principle (Wang [17]). Finally, Section 4 concludes. For any function g, by $g'(u)$ we mean the first derivative of g with respect to u.

2 The MEG distribution

The EG distribution with parameters $\beta > 0$ and $0 < p < 1$, as proposed by Adamidis and Loukas [1] has the following cumulative distribution function (CDF):

$$F_{EG}(x \mid \beta, p) = (1 - e^{-\beta x})(1 - pe^{-\beta x})^{-1};\ x > 0,\ \beta > 0,\ 0 < p < 1. \tag{2.1}$$

Introducing $\theta \in \Re = (-\infty, \infty)$ as the third parameter and using simple algebra in (2.1), CDF of the MEG distribution is obtained as

$$F(x; \theta, \beta, p) = \begin{cases} \left(1 - (1 - \theta\beta x)^{1/\theta}\right)\left(1 - p\,(1 - \theta\beta x)^{1/\theta}\right)^{-1} & \text{if } \theta \neq 0 \\ (1 - e^{-\beta x})(1 - pe^{-\beta x})^{-1} & \text{if } \theta = 0. \end{cases} \tag{2.2}$$

The support of the random variable (rv) X of the MEG distribution in (2.2) is $(0, \infty)$ when $\theta \leq 0$, and $\left(0, \frac{1}{\theta\beta}\right)$ when $\theta > 0$. The PDF of the MEG distribution with parameters θ, β, p, denoted by MEG(θ, β, p) is given by

$$f(x; \theta, \beta, p) = \begin{cases} \beta(1 - p)\,(1 - \theta\beta x)^{(1/\theta)-1}\left(1 - p\,(1 - \theta\beta x)^{1/\theta}\right)^{-2} & \text{if } \theta \neq 0 \\ \beta(1 - p)e^{-\beta x}(1 - pe^{-\beta x})^{-2} & \text{if } \theta = 0. \end{cases} \tag{2.3}$$

Remark 2.1 *The EG distribution is a limiting special case of the MEG distribution when $\theta \to 0$.* □

Remark 2.2 *The Exponential distribution is a limiting special case of the MEG distribution when $\theta \to 0$ and $p \to 0+$.* □

2.1 Statistical and reliability properties

Different properties of the MEG distribution are studied in this section. To be specific, we study the behaviour of the density function and the hazard rate function in detail. Moments of the distribution are also derived with some findings on the skewness and the kurtosis. Bowley's measure of skewness (S_k) is also computed with CV. Moreover, expressions of reversed hazard rate function and mean residual function are also derived.

The following theorem shows that the three-parameter MEG distribution as give in (2.3) takes various shapes for different choices of θ and p. Figure 1 shows the different shapes of the density functions for different choices of the parameters. Below we write $f(x)$ to mean $f(x; \theta, \beta, p)$.

Theorem 2.1 *The PDF of MEG(θ, β, p) distribution*

(i) is strictly decreasing when $-1 < \theta < 0$ and $p \in (0, 1)$;

(ii) does not exist when $\theta \leq -1$ and $p \in (0, 1)$;

Figure 1: PDF of MEG distribution

(iii) is strictly decreasing when $0 < \theta < 1$ *and* $p \in (0,1)$*;*

(iv) is strictly increasing when $\theta > 1$ *and* $p \leq \frac{\theta-1}{\theta+1}$*, and U-shaped for* $\theta > 1$ *and* $p \geq \frac{\theta-1}{\theta+1}$*.*

Proof. Assuming $u = (1-\theta\beta x)^{1/\theta}$, the first expression of (2.3) can be written as

$$\frac{f(x)}{\beta(1-p)} = u^{1-\theta}(1-pu)^{-2} = a(u), \text{ say.}$$

Now, differentiating $a(u)$ with respect to u, we get

$$a'(u) = u^{-\theta}(1-pu)^{-2}\left(1-\theta+2up(1-pu)^{-1}\right),$$

which gives

$$a'(u) \gtreqless 0 \text{ if and only if } u \gtreqless \frac{\theta-1}{p(1+\theta)} \tag{2.4}$$

Case I: $-1 < \theta < 0$. Then $u > 0$ and $\frac{\theta-1}{p(1+\theta)} < 0$. Thus, from (2.4) we see that $a'(u) < 0$ is not possible. This gives that $a'(u) \geq 0$ if and only if $u \geq \frac{\theta-1}{p(1+\theta)}$ (< 0), *i.e.*, if $u \geq 0$. So, $a(u)$ is increasing in u or $f(x)$ is decreasing in x, for all x. Therefore, the PDF of the MEG distribution is decreasing for $-1 < \theta < 0$, for all $p \in (0,1)$.

Case II: $\theta < -1$.
Here, $u > 0$ and $\frac{\theta-1}{p(1+\theta)} > 0$. Now,

$$u \gtreqless \frac{\theta-1}{p(1+\theta)} \text{ if and only if } x \lesseqgtr \frac{1}{\alpha\beta}\left[\left(\frac{p(\alpha-1)}{\alpha+1}\right)^{\alpha} - 1\right],$$

where $\theta = -\alpha; \alpha > 1$. But $\left(\frac{p(\alpha-1)}{\alpha+1}\right)^{\alpha} - 1 < 0$. Hence, $a'(u) > 0$ is not possible. Therefore, $a'(u) < 0$ if and only if $u < \frac{\theta-1}{p(1+\theta)}$ (> 0). So, $a(u)$ is decreasing in u or $f(x)$ is increasing in x, for all $x > 0$. But, PDF cannot be increasing in an infinite support. Hence, $\theta < -1$ does not make (2.3) a proper PDF.

Case III: $0 < \theta < 1$. Note that $\theta > 0$ gives $0 \le x \le \frac{1}{\theta\beta}$ which in turn implies $u \ge 0$ and $\frac{\theta-1}{p(\theta+1)} < 0$. Hence, $a'(u) > 0$ for $u > 0$ implying that $a(u)$ is increasing in u or $f(x)$ is decreasing in x. Therefore MEG distribution is strictly decreasing for $0 < \theta < 1$ and for all $p \in (0, 1)$.

Case IV: Let $\theta > 1$. From (2.4), we have that $a(u)$ is increasing in u if and only if $u \ge \frac{\theta-1}{p(1+\theta)}$. Now,

$$u \ge \frac{\theta-1}{p(1+\theta)} \text{ if and only if } x \le \frac{1}{\theta\beta}\left[1 - \left(\frac{\theta-1}{p(\theta+1)}\right)^{\theta}\right] = x_0, \text{ say.}$$

Again,

$$\left[1 - \left(\frac{\theta-1}{p(\theta+1)}\right)^{\theta}\right] \geqslant 0, \text{ if and only if } p \ge \frac{\theta-1}{\theta+1}.$$

Hence, $a(u)$ is increasing in u, or, equivalently,

$$f(x) \text{ is decreasing in } x, \text{ for } p \ge \frac{\theta-1}{\theta+1}, \text{ and } 0 \le x \le x_0. \tag{2.5}$$

Similarly, $a(u)$ is decreasing in u if

$$\frac{1}{\theta\beta}\left[1 - \left(\frac{\theta-1}{p(\theta+1)}\right)^{\theta}\right] \le x \le \frac{1}{\theta\beta}.$$

Now, $1 - (\frac{\theta-1}{p(\theta+1)})^{\theta} \gtreqless 0$ when $p \gtreqless \frac{\theta-1}{\theta+1}$. Therefore, for $p \ge \frac{\theta-1}{\theta+1}$,

$$f(x) \text{ is increasing in } x \ge x_0 \tag{2.6}$$

and, for $p \le \frac{\theta-1}{\theta+1}$,

$$f(x) \text{ is increasing in } x \ge 0 \tag{2.7}$$

Combining (2.5) and (2.6), we conclude that $f(x)$ takes U-shape when $\theta > 1$ and $p \ge \frac{\theta-1}{\theta+1}$. Moreover, (2.7) confirms that $f(x)$ is increasing when $\theta > 1$ and $p \le \frac{\theta-1}{\theta+1}$. □

Remark 2.3 *From (2.2), it is easy to derive that the $\xi^{th}(\xi \in [0, 1])$ order quantile, say x_ξ, of the MEG distribution can be obtained by solving $F(x_\xi; \theta, \beta, p) = \xi$, which gives the ξ^{th} order quantile of MEG distribution as*

$$x_\xi = \begin{cases} \frac{1}{\theta\beta}\left[1 - \left(\frac{1-\xi}{1-p\xi}\right)^{\theta}\right] & \text{if } \theta \neq 0 \\ -\frac{1}{\beta}\log\left(\frac{1-\xi}{1-p\xi}\right) & \text{if } \theta = 0. \end{cases} \tag{2.8}$$

Median and other percentiles of the MEG distribution can be obtained from (2.8). □

The rth order raw moment, μ'_r, and hence the expectation and the variance of the MEG distribution can be obtained from the following theorem.

Theorem 2.2 *For $\theta > 0$, $E\left(1 - \theta\beta X\right)^r = \frac{1-p}{p}\sum_{i=1}^{\infty}\frac{ip^i}{r\theta+i}$.*

Proof. Using the PDF in (2.3) of MEG distribution, we obtain

$$\begin{aligned} E(1-\theta\beta X)^r &= \beta(1-p)\int_S (1-\theta\beta x)^{r+\frac{1}{\theta}-1}\left(1-(p(1-\theta\beta x)^{1/\theta}\right)^{-2}dx \\ &= \frac{1-p}{p}\int_0^p t^{r\theta}(1-t)^{-2}dt \\ &= \frac{1-p}{p^{r\theta+1}}\int_0^p\left(\sum_{i=1}^{\infty} i t^{i-1}\right)t^{r\theta}dt \\ &= \frac{1-p}{p^{r\theta+1}}\sum_{i=1}^{\infty}\frac{ip^{r\theta+i}}{r\theta+i} \\ &= \frac{1-p}{p}\sum_{i=1}^{\infty}\frac{ip^i}{r\theta+i}. \end{aligned}$$

In the first equality, S is the support (based on whether $\theta \le 0$ or $\theta > 0$), whereas in the second equality, the transformation $t = p(1-\theta\beta x)^{1/\theta}$ is used. □

Remark 2.4 *In order for the rth moment of the MEG distribution to exist we must have $\theta > -\frac{1}{r}$. Hence, for the MEG distribution, all moments exist for $\theta > 0$.*

Corollary 2.1 *Expectation and variance of the MEG distribution are as under.*

$$\begin{aligned} \mu &= \frac{1}{\theta\beta}\left[1-\frac{1-p}{p}\sum_{j=1}^{\infty}\frac{jp^j}{\theta+j}\right] \\ \mu_2 &= \frac{1}{\theta^2\beta^2}\left[\frac{1-p}{p}\sum_{i=1}^{\infty}\frac{ip^i}{2\theta+i}+2\theta\beta\mu-1\right]-\mu^2. \end{aligned}$$

Remark 2.5 *The central moment (μ_r) can be easily derived from Theorem 2.2 and hence moment measure of skewness ($\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}$) and kurtosis ($\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$) along with CV. Table 1 shows values of moments, variance, γ_1, γ_2 and CV along with quartiles and Bowley's measure of skewness for a few choices of parameters of the MEG distribution.* □

Survival function of the MEG distribution is given by

$$\bar{F}(x;\theta,\beta,p) = \begin{cases} (1-p)\,(1-\theta\beta x)^{1/\theta}\left(1-p\,(1-\theta\beta x)^{1/\theta}\right)^{-1} & \text{if } \theta \neq 0 \\ (1-p)e^{-\beta x}(1-pe^{-\beta x})^{-1} & \text{if } \theta = 0, \end{cases}$$

with hazard rate

$$h(x) = \begin{cases} \beta\,(1-\theta\beta x)^{-1}\left(1-p\,(1-\theta\beta x)^{1/\theta}\right)^{-1} & \text{if } \theta \neq 0 \\ \beta(1-pe^{-\beta x})^{-1} & \text{if } \theta = 0, \end{cases} \tag{2.9}$$

and reversed hazard rate

$$\widetilde{h}(x) = \begin{cases} \beta(1-p)\,(1-\theta\beta x)^{\frac{1}{\theta}-1}\left[\left(1-(1-\theta\beta x)^{1/\theta}\right)\left(1-p\,(1-\theta\beta x)^{1/\theta}\right)\right]^{-1} & \text{if } \theta \neq 0 \\ \beta(1-p)e^{-\beta x}\left[\left(1-e^{-\beta x}\right)\left(1-pe^{-\beta x}\right)\right]^{-1} & \text{if } \theta = 0. \end{cases}$$

Figure 2: Hazard rate of MEG distribution

The following theorem gives us a general result on hazard rate function of the MEG distribution. It shows that the distribution has decreasing, increasing and bathtub-shaped failure rates. This fact has been depected through Figure 2 for different choices of the parameters.

Theorem 2.3 *The hazard rate function of MEG(θ, β, p) distribution is*

(i) strictly decreasing when $-1 < \theta < 0$ and $p \in (0,1)$;

(ii) strictly increasing for $\theta > 0$ and $p \leq \frac{\theta}{\theta+1}$, and bathtub-shaped for $\theta > 0$ and $p \geq \frac{\theta}{\theta+1}$.

Proof. Assuming $u = (1-\theta\beta x)^{1/\theta}$ as before, (2.9) can be written, for $\theta \neq 0$, as

$$\left[\frac{h(x)}{\beta}\right]^{-1} = u^{\theta}(1-pu) = b(u), say.$$

Then, $b'(u) = u^{\theta-1}\left(\theta - pu(1+\theta)\right)$. Now,

$$b'(u) \gtreqless 0 \text{ if and only if } u \lesseqgtr \frac{\theta}{p(1+\theta)}. \tag{2.10}$$

Case I: $-1 < \theta < 0$. Then $u > 0$ and $\frac{\theta}{p(1+\theta)} < 0$. Thus, from (2.10), we have that $b'(u) > 0$ is not possible. This gives that $b'(u) \leq 0$ if and only if $u \geq \frac{\theta}{p(1+\theta)} (< 0)$, *i.e.*, if $u \geq 0$. So, $b(u)$ is decreasing in u, or equivalently, $h(x)$ is decreasing in $x \geq 0$. Therefore, the MEG distribution has decreasing failure rate for $-1 < \theta < 0$, for all $p \in (0,1)$.

Case II: $\theta > 0$. From (2.10), we have that $b(u)$ is increasing in u if and only if $u \leq \frac{\theta}{p(1+\theta)}$. Now,

$$u \leq \frac{\theta}{p(1+\theta)} \text{ if and only if } x_0 = \frac{1}{\theta\beta}\left[1 - \left(\frac{\theta}{p(\theta+1)}\right)^{\theta}\right] \leq x \leq \frac{1}{\theta\beta}.$$

Again, $1 - \left(\frac{\theta}{p(\theta+1)}\right)^{\theta} \gtreqless 0$ if and only if $p \gtreqless \frac{\theta}{\theta+1}$. Hence, $b(u)$ is increasing in u, or equivalently,

$$h(x) \text{ is increasing in } x, \text{ when } p \leq \frac{\theta}{\theta+1}. \tag{2.11}$$

Similarly, $b(u)$ is increasing in u when $1-(\frac{\theta}{p(\theta+1)})^{\theta}\geq 0$, *i.e.*, when $p\geq\frac{\theta}{\theta+1}$. Therefore,

$$h(x) \text{ is increasing in } x, \text{ for } p\geq\frac{\theta}{\theta+1}, \text{ and } x_0\leq x\leq\frac{1}{\theta\beta}. \tag{2.12}$$

Again, $b(u)$ is decreasing in u when $u\geq\frac{\theta}{p(\theta+1)}$, *i.e.*, when $x\leq x_0$. However, as we have seen from the previous discussion, $x_0\geq 0$ if $p\geq\frac{\theta}{\theta+1}$. Therefore,

$$h(x) \text{ is decreasing in } x, \text{ for } 0\leq x\leq x_0, \text{ when } p\geq\frac{\theta}{\theta+1}. \tag{2.13}$$

Combining (2.12) and (2.13), we observe that the failure rate of MEG distribution is bathtub-shaped when $p\geq\frac{\theta}{\theta+1}$. Also, (2.11) ensures that the distribution has increasing failure rate when $p\leq\frac{\theta}{\theta+1}$. □

Remark 2.6 *Theorem 2.1 (ii) proves that the hazard rate function of the MEG distribution does not exist for* $\theta<-1$. □

Next, we derive the expression for mean residual life function (MRL) of the MEG distribution. The proof is similar to that of Theorem 2.2.

Theorem 2.4 *Mean residual function of the MEG distribution is given by*

$$E\left(X-x_0\mid X\geq x_0\right)=\frac{1-p}{\theta\bar{F}(x_0)}\left[1-\frac{1}{\beta p}\sum_{i=1}^{\infty}\frac{ip^i(1-\theta\beta x_0)^{\frac{i}{\theta}+1}}{\theta+i}-\theta(1-\theta\beta x_0)^{\frac{1}{\theta}}\right]$$

.

2.2 Estimation of the parameters

Here, we consider estimation of the unknown parameters of the MEG distribution by the method of maximum likelihood. Let $x_1,x_2,...,x_n$ be a random sample of size n drawn from (2.3) with parameters $\mathbf{\Psi}=(\theta,\beta,p)$. Then the log-likelihood function $L(\Psi)$ for MEG distribution can be written as

$$L(\mathbf{\Psi})=n\log\beta+n\log(1-p)+\left(\frac{1}{\theta}-1\right)\sum_{i=1}^{n}\log\left(1-\theta\beta x_i\right)-2\sum_{i=1}^{n}\log\left(1-p(1-\theta\beta x_i)^{1/\theta}\right). \tag{2.14}$$

The likelihood equations are given by

$$\left.\begin{aligned}\frac{\partial L}{\partial\beta}&=\frac{n}{\beta}-(1-\theta)\sum_{i=1}^{n}\frac{x_i}{A_i}-2p\sum_{i=1}^{n}\frac{C_ix_i}{A_i(1-pC_i)}\\ \frac{\partial L}{\partial\theta}&=-1/\theta^2\sum_{i=1}^{n}B_i-\beta\left(\frac{1}{\theta}-1\right)\sum_{i=1}^{n}\frac{x_i}{A_i}+\frac{-2p}{\theta}\sum_{i=1}^{n}\left[\frac{C_i}{1-pC_i}\left(\frac{B_i}{\theta}+\frac{\beta x_i}{A_i}\right)\right]\\ \frac{\partial L}{\partial p}&=\frac{-n}{1-p}+2\sum_{i=1}^{n}\frac{C_i}{1-pC_i}\end{aligned}\right\} \tag{2.15}$$

where $A_i = 1 - \theta\beta x_i$; $B_i = \log A_i$; $C_i = A_i^{1/\theta}$. The maximum likelihood estimator (MLE) of $\mathbf{\Psi}$, say $\hat{\mathbf{\Psi}}$, is the simultaneous solutions of the likelihood equations (2.15) when $\theta < 0$.

Estimation for the case of $\theta > 0$ is somewhat different as the support of the distribution is finite and depends on the unknown parameters θ and β. Let us propose a reparametrization of β, θ, p as (α, θ, p) where $\alpha = 1/(\theta\beta)$. Hence, (2.3) can be rewritten as

$$f(x;\theta,\alpha,p) = \frac{1-p}{\theta\alpha}\left(1-\frac{x}{\alpha}\right)^{(1/\theta)-1}\left(1-p\left(1-\frac{x}{\alpha}\right)^{1/\theta}\right)^{-2} ;\ 0 < x < \alpha. \tag{2.16}$$

Based on a random sample from (2.16), the MLE's of (α, θ, p) are obtained by maximizing the log-likelihood function

$$\begin{aligned} L_1(\alpha,\theta,p) = n\log(1-p) - n\log\alpha - n\log\theta + \\ \left(\frac{1}{\theta}-1\right)\sum_{i=1}^{n}\log\left(1-\frac{x_{(i)}}{\alpha}\right) - 2\sum_{i=1}^{n}\log\left(1-p\left(1-\frac{x_{(i)}}{\alpha}\right)^{1/\theta}\right). \end{aligned} \tag{2.17}$$

Here, $x_{(i)}$ denotes the ith ordered observation. It is obvious from (2.17) that, for $0 < p < 1$, as $\alpha \downarrow x_{(n)}$ (the largest observation among $x_i, i = 1, 2, ..., n$), $L_1(\alpha, \theta, p) \to \infty$ (resp. $-\infty$) when $\theta > 0$ (resp. $\theta \in (0, 1)$), resulting in the non-existence of MLE's.

The most natural way (Smith [15]) to estimate the parameters to handle the situation is to estimate α first by its consistent estimator $\hat{\alpha} = x_{(n)}$. The modified log-likelihood function based on the remaining $(n-1)$ observations after ignoring $x_{(n)}$ and substituting α as $x_{(n)}$ is given as

$$\begin{aligned} L_2(\theta,p) = (n-1)\log(1-p) - (n-1)\log x_{(n)} - (n-1)\log\theta + \\ \left(\frac{1}{\theta}-1\right)\sum_{i=1}^{n-1}\log\left(1-\frac{x_{(i)}}{x_{(n)}}\right) - 2\sum_{i=1}^{n-1}\log\left(1-p\left(1-\frac{x_{(i)}}{x_{(n)}}\right)^{1/\theta}\right). \end{aligned} \tag{2.18}$$

Likelihood equations from (2.18) are derived as

$$\left.\begin{aligned} \frac{\partial L_2}{\partial\theta} &= -(n-1)/\theta - 1/\theta^2\sum_{i=1}^{n-1}E_i - 2p/\theta^2\sum_{i=1}^{n-1}\frac{E_iF_i}{1-pF_i} \\ \frac{\partial L_2}{\partial p} &= \frac{-(n-1)}{1-p} + 2\sum_{i=1}^{n-1}\frac{F_i}{1-pF_i} \end{aligned}\right\} \tag{2.19}$$

where $D_i = 1 - \frac{x_{(i)}}{x_{(n)}}$; $E_i = \log D_i$; $F_i = D_i^{1/\theta}$. Since no closed form expressions for the MLEs of $\mathbf{\Psi}$ are available from the expressions (2.15) and (2.19), we go for detailed simulation studies to capture the means (μ) and the standard deviations (SD) of the MLEs of $\mathbf{\Psi}$. Ten thousand replicates of Monte-Carlo experiments of size 20, 50, 100 and 500 are considered in the present investigation for each of the six sets of $\mathbf{\Psi}$ viz. (-0.5,0.5,0.5), (-5.0,0.5,0.5), (0.5,0.5,0.5), (0.5,5.0,0.5), (5,0.5,0.5), (5.0,5.0,0.5). These estimates are obtained by using the function *optim* from the statistical software *R* (version R.3.0.1). In the current context, we recommend to use quasi-Newton algorithms, namely the BFGS algorithm for numerical maximization of log-likelihood functions. The results from simulated data sets are reported in Table 2. The results show that the estimates are quite stable around the assumed values of $\mathbf{\Psi}$ and moreover, standard errors of the MLEs decrease when sample size increases, which is expected.

2.3 An application of MEG distribution in reliability

Lifetime data modeling in the literature of reliability analysis is studied extensively by several researchers for different life distributions which are mainly based on some modifications and generalizations of exponential or Weibull distributions. While this modification is carried out in some of the life distributions through exponentiation and its extension viz. Exponentiated or Generalized Exponential (GE) distribution (e.g. Gupta and Kundu [8], Kundu and Gupta[10]), Exponentiated Weibull (EW) distribution (e.g. Mudholkar and Srivastava [12]), there are others where lifetime distributions are compounded with distribution of unknown number of components yielding a new class of life distributions viz. Exponential-Geometric (EG) distribution (Adamidis and Loukas [1]), Exponential-Poisson (EP) distribution (Kus [11]), Exponential-Logarithmic (EL) distribution (Tahmasbi and Rezaei [16]), Weibull-Geometric (WG) distribution (Barreto-Souza et al. [2]) and so on.

In this section, we fit the MEG distribution to a real data set from Proschan [14]. The data set consists of the number of successive failures for the air conditioning system of each member in a fleet of 13 Boeing 720 jet airplanes. The pooled data with 214 observations were first analyzed by Proschan [14] and discussed further by Dahiya and Gurland [3] and Gleser [6]. To carry out the comparison of the performance of our proposed model, we have considered some alternative models as discussed in the earlier paragraph viz. EG, EP, EL and WG which also fit the same data set from Proshan [14]. For the data set that we consider here, we derive the maximum likelihood estimates, Kolmogorov-Smirnov (K-S) statistic and the corresponding p-value for each of the distributions. The results of the data analysis are shown in Table 3. Moreover, we have conducted chi-square goodness-of-fit test to the data set and compared observed and expected frequencies for each of the distributions. These results are demonstrated in Table 4.

The results from Table 3 show that the K-S test statistic and the p-value for the proposed MEG model take the smallest and the largest values respectively for the data set as compared to the other models, ensuring its applicability in practice. These results are further validated by the comparison between observed and expected frequencies in Table 4, where expected frequencies for the MEG model commensurate well with the observed frequencies, also showing the highest p-value. In fact, the performance of all the models mentioned here along with the proposed one are quite good in terms of K-S and p-value.

The Quantile-Quantile (Q-Q) plots are also shown in Figure 3. As the figures show, all the models are very similar and provide good fits to the data set. The proposed model offers an

Figure 3: Q-Q plots

attractive alternative to these well-established models not only to analyze the data set, but also for its flexibility and potentiality with respect to shape and hazard rates.

3 Beta-equivalent MEG distribution

Here we transform the MEG distribution to a beta-equivalent distribution with support $(0,1)$ for $\theta > 0$, which is named as beta-equivalent MEG distribution and is denoted by MEG_b. If X follows the MEG distribution as given in (2.3), then $U = X\theta\beta$ has the MEG_b distribution whose PDF is given by

$$f_U(x \mid \theta, p) = \frac{1-p}{\theta}\,(1-x)^{\frac{1}{\theta}-1}\left(1-p\,(1-x)^{1/\theta}\right)^{-2};\ 0<x<1,\ \theta>0,\ 0<p<1, \tag{3.1}$$

with CDF

$$F_U(x \mid \theta, p) = \left(1-(1-x)^{1/\theta}\right)\left(1-p\,(1-x)^{1/\theta}\right)^{-1}. \tag{3.2}$$

Remark 3.1 *Being a two-parameter scale-transformed distribution, all the statistical and the reliability properties of the MEG_b distribution related to the shape of the pdf and the hazard rate function remain same as the MEG distribution.* □

Remark 3.2 *We intend to find out the type of Pearsonian system of curves the MEG_b distribution belongs to. For this purpose, we have computed b_0, b_1 and b_2, and hence $\kappa = \frac{b_1^2}{4b_0b_2}$ for different choices of the parameters and shown in Table 5. Details on Pearsonian system of curves can be obtained in Elderton and Johnson [5]. The κ-criterion suggests that the distribution belongs to Pearson's Type I system of curves where beta distribution also belongs to.* □

3.1 Use of MEG_b distribution in insurance

Traditionally, an insurance risk X is defined as a non-negative loss random variable with CDF G_X and survival function (also known as decumulative distribution function in the actuarial literature) S_X, and a premium calculation principle refers to a functional $\rho:\ X \to [0,\infty)$. The premium principle $\rho(X)$ gives the premium associated with the contract providing coverage against

X. For an overview of premium principle, see Denuit et al. [4]. In general, for a risk X, the expected loss can be evaluated directly from its survival function as $\rho_1^*(X) = E(X) = \int_0^\infty S_X(x)dx$ and is commonly applied when decision-makers agree on the risk distribution. Note that $\rho_1^*(X)$ is the simplest premium principle and is known as the net premium. As there does not exist any unique risk distribution, insurers add a loading to X that reflects the danger associated with the risk. Premium principle by Wang [17] suggests to transform the survival function by a continuous and non-decreasing distortion function $h : [0, 1] \to [0, 1]$ with $h(0) = 0$ and $h(1) = 1$. The distortion function $h(S_X(.))$ can be thought of as a risk-adjusted survival function of the random variable X_h (say). A distortion risk measure associated with distortion function h, for a random loss X, is given by $\rho_h(X) = E_h(X) = \int_0^\infty h(S_X(x))dx$. The distortion risk measure adjusts the true probability measure to give more weight to higher risk events. Both of the quantile and the conditional tail expectation (CTE) risk measures fall into the class of distortion risk measures. They are by far the most commonly used distortion measures for capital adequacy, but others are also seen in practice, particularly for premium setting in property and casualty insurance.

A concave distortion function gives more weight to higher risk events. For instance, Wang [17] suggests to use $\rho_h(X)$ as a premium principle; for insurance premium purpose, $\rho_h(X)$ must be at least equal to $\rho_1^*(X)$ and such is the case when h is concave. Moreover, as h is increasing and concave, $h(x) \geq x$ for all $x \in (0, 1)$ which ensures that $h(S(x)) \geq S(x)$ for all x, or equivalently, $X \leq_{st} X_h$, resulting in $\rho_h(X) \geq \rho_1^*(X)$. It is to be mentioned here that, for two random variables X and Y with respective survival functions $\bar{F}$ and $\bar{G}$, X is said to be smaller that Y in usual stochastic order, written as $X \leq_{st} Y$, if $\bar{F}(x) \leq \bar{G}(x)$, for all x. Hence, the premium principle contains a non-negative loading. In fact, the premium principle with concave distortion function satisfies some desirable properties of premium functional viz. non-ripoff, positive homogeneity, comonotonicity and subadditivity. These four axioms make the concave distortion risk measure coherent; for more details, see Denuit et al. [4].

Here we use the CDF of the MEG_b distribution to distort the survival function S_X of any loss random variable to offer a premium with non-negative loading. Next, we intend to find out another risk-adjusted premium principle, known as dual power premium principle (Wang [17]). It can be easily shown that the concave distortion function $h(x) = \frac{1-(1-x)^{1/\theta}}{1-p(1-x)^{1/\theta}}$ (See Theorem 3.1) transforms the survival function $S_X(x)$ of loss random variable into the survival function $1 - [G_X(x)]^n$ which corresponds to the survival function of the random variable $X_{n:n}$, the nth order statistic, where $X_i,\ i = 1, 2, ..., n$ are iid random variables. So, the corresponding risk-adjusted premium is $\rho_n^*(X) = E(X_{n:n})$. It is obvious that $\rho_1^*(X) \leq \rho_n^*(X)$.

The distortion function, the CDF of the MEG_b distribution in the present case, is said to follow the dual power premium principle if $\rho_h(X) \leq \rho_n^*(X)$ for some relationship between the parameters of the distortion function, the risk distribution and the sample size n. The results in Table 6 confirm that the premium obtained by distorting original loss distribution by the MEG_b distribution lies between the net premium and the dual power premium

$(\rho_1^*(X) \leq \rho_h(X) \leq \rho_n^*(X))$ for selected choices of the parameters. However, it is also observed from Table 7 that the same distortion function does not satisfy the dual power premium principle for some other choices of the parameters with the same sample size. We have considered the exponential, the Weibull, the lognormal and the inverse Gaussian distributions as the original loss distribution with different choices of the parameters. Hence, the upper bound of the distorted premium principle with CDF of MEG_b distribution as the distortion function may not always be the dual premium principle. We conclude this section by showing that the CDF of the MEG_b distribution is concave.

Theorem 3.1 *The CDF of the MEG_b distribution (given in (3.2)) is concave for $\theta \leq 1$.*

Proof. Differentiating (3.2) twice, we get

$$F''(x) = -\left(\frac{1-p}{\theta}\right)\left(1-p\,(1-x)^{1/\theta}\right)^{-3}(1-x)^{\frac{1}{\theta}-2}\left[p\,(1-x)^{1/\theta}\left(\frac{1}{\theta}+1\right)+\left(\frac{1}{\theta}-1\right)\right].$$

For $x \in (0,1)$, $0 < p < 1$ and $\theta > 0$, it is obvious that $F''(x)$ is concave for $0 < \theta \leq 1$. □

4 Conclusion

In this paper a new PDF with both unbounded and bounded support is proposed, which exhibits a variety of shapes of PDF and hazard functions. The new distribution, called modified EG distribution is derived from the exponential-geomeric distribution, introduced by Adamidis and Loukas [1]. The parameters of the proposed distribution is estimated using the maximum likelihood method through Monte-Carlo simulation. The distribution with scale-transformed bounded support on $(0,1)$, known as beta-equivalent MEG distribution is shown to belong to Pearsonian Type I system of curves and is suggested as a special class of distorted premium principle in the insurance context. The proposed distribution with unbounded support is considered as a competitive lifetime model with respect to some well-established models.

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(θ, β, p)	μ	μ_2	CV	γ_1	γ_2	Q_1	Q_2	Q_3	S_k
(-0.10,2,0.2)	0.493	0.338	1.179	3.014	16.900	0.120	0.303	0.651	0.311
(-0.10,2,0.6)	0.332	0.216	1.400	3.785	26.209	0.063	0.171	0.410	0.378
(-0.24,2,0.2)	0.578	0.709	1.456	6.743	513.567	0.122	0.316	0.711	0.341
(-0.24,2,0.6)	0.380	0.422	1.710	8.151	736.727	0.064	0.175	0.434	0.400
(0.4,0.5,0.2)	1.298	1.067	0.795	0.212	-0.076	0.451	1.048	1.935	0.195
(0.4,0.5,0.6)	0.935	0.827	0.973	1.393	1.570	0.244	0.630	1.352	0.303
(0.4,2.0,0.2)	0.325	0.067	0.795	0.881	0.063	0.113	0.262	0.484	0.197
(0.4,2.0,0.6)	0.234	0.052	0.972	1.393	1.570	0.061	0.157	0.338	0.307
(2.0,0.5,0.2)	0.629	0.093	0.486	-0.039	-1.487	0.377	0.691	0.913	-0.172
(2.0,0.5,0.6)	0.505	0.099	0.622	0.010	-1.110	0.221	0.490	0.793	0.059
(2.0,2.0,0.2)	0.157	0.006	0.486	-0.461	-1.084	0.094	0.173	0.228	-0.179
(2.0,2.0,0.6)	0.126	0.006	0.622	0.064	-1.317	0.055	0.122	0.198	0.063

Table 1: Moments and quartiles of the MEG distribution for some choices of (θ, β, p)

(θ, β, p)	n	MLE			SD		
(-0.5,0.5,0.5)	20	0.55857	-0.55471	0.59873	0.0044687	0.0040145	0.0074738
	50	0.54524	-0.57388	0.56295	0.0005057	0.0010094	0.0005307
	100	0.52891	-0.54595	0.54339	0.0004016	0.0004605	0.0002835
	500	0.49229	-0.52165	0.53313	0.0003393	0.0002993	0.0002079
(-5.0,0.5,0.5)	20	5.59075	-0.57812	0.57266	0.0446627	0.0045076	0.0163101
	50	5.49697	-0.60776	0.55305	0.0032612	0.0023078	0.0006664
	100	5.50101	-0.62647	0.51458	0.0026782	0.0016946	0.0004795
	500	5.33426	-0.64013	0.48321	0.0020705	0.0008048	0.0001576
(0.5,0.5,0.5)	20	0.61561	0.58036	0.63011	0.0225243	0.0898215	0.0211134
	50	0.58632	0.54686	0.60020	0.0086874	0.0436351	0.0056752
	100	0.55655	0.51928	0.56231	0.0009265	0.0052417	0.0007985
	500	0.51100	0.49685	0.52332	0.0005563	0.0007372	0.0002322
(0.5,5.0,0.5)	20	5.10213	0.65178	0.58390	0.0721342	0.0560551	0.0261122
	50	5.03214	0.59113	0.56662	0.0083615	0.0053224	0.0063531
	100	4.98201	0.49174	0.50888	0.0034747	0.0002398	0.0003215
	500	4.91201	0.46961	0.51030	0.0002983	0.0001531	0.0001555
(5.0,0.5,0.5)	20	0.70077	5.01690	0.51395	0.0483501	0.0029333	0.0175577
	50	0.61212	5.10325	0.52101	0.0232521	0.0012035	0.0096321
	100	0.54331	5.00692	0.51115	0.0088524	0.0005664	0.0005036
	500	0.51655	4.95983	0.50601	0.0008871	0.0003134	0.0002691
(5.0,5.0,0.5)	20	6.76976	5.16878	0.52539	0.4781179	0.0066869	0.0186187
	50	5.01394	4.97019	0.59852	0.0024631	0.0012543	0.0005525
	100	4.90802	4.94170	0.60559	0.0017124	0.0006001	0.0004561
	500	4.99433	4.91830	0.61991	0.0006238	0.0003339	0.0001983

Table 2: MLEs and SD of parameters of MEG Distribution based on Monte Carlo Simulation

Distribution	Estimates	K-S	p-value
MEG	(-0.1003;0.0106;0.1936)	0.0416	0.8546
WG	(0.0051;1.1843;0.7695)	0.0658	0.5021
EG	(0.0082;0.4088)	0.0492	0.6302
EP	(0.0079;1.2011)	0.0471	0.7623
EL	(0.0086;0.4005)	0.0486	0.6734

Table 3: ML estimates, K-S statistics and p-values for the Proschan data

Class Intervals	Observed frequency	Expected frequency				
		MEG	WG	EG	EP	EL
0-50	99	97.31	94.39	98.11	98.40	97.61
50-100	51	48.38	48.84	47.03	47.76	46.92
100-150	19	26.36	28.47	25.87	25.82	26.18
150-200	14	15.21	16.39	15.31	15.01	15.61
200-250	14	9.14	9.53	9.45	9.16	9.64
250-300	4	5.66	5.66	5.99	5.78	6.08
300-350	3	3.60	3.44	3.86	3.73	3.88
350-400	2	2.33	2.14	2.51	2.44	2.49
400-450	3	1.54	1.36	1.65	1.61	1.61
450-500	2	1.03	0.88	1.09	1.07	1.04
500-550	1	0.70	0.58	0.72	0.72	0.68
550-600	0	0.49	0.39	0.48	0.48	0.44
600-	1	1.24	0.91	0.94	0.99	0.82
	p-value	0.6507	0.4685	0.6394	0.6292	0.6106

Table 4: Observed and expected frequencies of the Proschan data

$\theta \downarrow$ $p \rightarrow$	0.1	0.3	0.5	0.7	0.9
0.1	-2.23 (1.6,3.1)	-2.81 (1.8,4.0)	-3.85 (2.1,5.5)	-6.31 (2.6,8.8)	-20.25 (4.0,22.8)
0.5	-0.16 (0.6,-0.5)	-0.25 (0.8,-1.0)	-0.43 (1.1,0.4)	-0.85 (1.5,1.7)	-2.83 (2.6,7.5)
0.9	-0.01 (0.2,-1.1)	-0.04 (0.3,-0.2)	-0.11 (0.6,-0.8)	-0.31 (0.9,-0.1)	-1.37 (1.9,3.5)
2.0	-0.09 (-0.5,-0.9)	-0.04 (-0.4,-1.2)	-0.02 (-0.1,-1.3)	-0.02 (0.3,-1.2)	-0.43 (1.2,0.3)
5.0	-0.48 (-1.3,0.4)	-0.26 (-0.9,-0.4)	-0.08 (-0.5,-1.1)	-0.85 (1.5,1.7)	-0.03 (0.4,-1.2)

Table 5: κ (γ_1, γ_2) values of the MEG_b distribution for different choices of parameters (θ, p)

Premium	$\rho_1^*(X)$	$\rho_h(X)$				$\rho_n^*(X)$	
Parameters of $MEG_b \rightarrow$		$\theta = 0.2$	$\theta = 0.4$	$\theta = 0.6$	$\theta = 0.8$	$n = 15$	$n = 25$
		$p = 0.1$	$p = 0.5$	$p = 0.7$	$p = 0.9$		
Loss density $\downarrow$							
Exponential							
$\lambda = 0.5$	0.500	1.176	1.066	1.073	1.379	1.659	1.908
$\lambda = 1.0$	1.000	2.352	2.132	2.146	2.759	3.318	3.816
$\lambda = 3.0$	3.000	7.056	6.396	6.439	8.279	9.955	11.447
Weibull							
$\lambda = 0.5, \gamma = 0.5$	1.000	3.524	3.112	3.221	4.959	6.296	8.084
$\lambda = 1.0, \gamma = 1.5$	0.903	1.719	1.591	1.589	1.896	2.192	2.414
$\lambda = 2.0, \gamma = 2.5$	1.774	2.729	2.576	2.876	2.876	3.180	3.375
$\lambda = 2.5, \gamma = 3.0$	2.232	3.229	3.084	3.069	3.368	3.674	3.863
lognormal							
$\mu = 0.5, \sigma = 0.50$	1.868	3.188	2.981	2.990	3.565	4.085	4.556
$\mu = 1.0, \sigma = 1.00$	2.718	11.516	10.387	10.598	14.836	18.192	22.369
$\mu = 2.0, \sigma = 1.50$	22.760	78.227	69.495	72.426	115.999	146.135	195.068
$\mu = 2.5, \sigma = 0.75$	16.139	34.304	31.401	31.734	41.100	49.035	57.515
Inverse Gaussian							
$\mu = 0.5, \sigma = 0.5$	0.500	1.156	1.050	1.063	1.407	1.703	2.011
$\mu = 1.0, \sigma = 1.0$	1.000	2.313	2.100	2.126	2.185	3.406	4.021
$\mu = 2.0, \sigma = 1.5$	2.000	4.982	4.495	4.569	6.221	7.614	9.133
$\mu = 2.5, \sigma = 2.0$	2.500	6.124	5.533	5.620	7.602	9.282	11.093

Table 6: $\rho_1^*(X)$, $\rho_h(X)$ and $\rho_n^*(X)$ for different loss PDF's with varying choices of parameters

Premium	$\rho_1^*(X)$	$\rho_h(X)$		$\rho_n^*(X)$	
Parameters of $MEG_b \rightarrow$		$\theta = 0.1$	$\theta = 0.2$	$n = 15$	$n = 25$
		$p = 0.8$	$p = 0.9$		
Loss density $\downarrow$					
Exponential					
$\lambda = 0.5$	0.500	2.007	2.038	1.659	1.908
$\lambda = 1.0$	1.000	4.154	4.076	3.318	3.816
$\lambda = 3.0$	3.000	12.462	12.229	9.955	11.447
Weibull					
$\lambda = 0.5, \gamma = 0.5$	9.786	9.578	3.112	6.296	8.084
$\lambda = 1.0, \gamma = 1.5$	2.546	2.507	1.591	2.192	2.414
$\lambda = 2.0, \gamma = 2.5$	3.478	3.440	2.576	3.180	3.375
$\lambda = 2.5, \gamma = 3.0$	3.958	3.921	3.368	3.674	3.863
lognormal					
$\mu = 0.5, \sigma = 0.50$	1.868	4.900	4.829	4.085	4.556
$\mu = 1.0, \sigma = 1.00$	2.718	26.387	25.886	18.192	22.369
$\mu = 2.0, \sigma = 1.50$	241.679	236.984	69.495	146.135	195.068
$\mu = 2.5, \sigma = 0.75$	16.139	64.662	63.517	49.035	57.515
Inverse Gaussian					
$\mu = 0.5, \sigma = 0.5$	0.500	2.248	2.205	1.703	2.011
$\mu = 1.0, \sigma = 1.0$	1.000	4.496	4.410	3.406	4.021
$\mu = 2.0, \sigma = 1.5$	2.000	10.345	10.139	7.614	9.133
$\mu = 2.5, \sigma = 2.0$	2.500	12.528	12.280	9.282	11.093

Table 7: $\rho_1^*(X)$, $\rho_h(X)$ and $\rho_n^*(X)$ for different loss PDF's with varying choices of parameters

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