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Predicting the Probability of Default Using Asset Correlation of a Loan Portfolio

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We use the asymptotic single risk factor model, which is a portfolio invariant model and preferred by BCBS with the factor based structural CreditMetrics portfolio default model to empirically estimate the Probability of default with asset correlation of a loan portfolio based on primary data from Public Sector Banks and compared the results with the estimated Probability of default without any asset correlation. We have used actual bank loan rating transition data for the period 2000-2010. Our study evidences that probability of default improves with asset correlation. We also find that asset correlation is an increasing function of probability of default. High rating firms have low correlation than low rating firms. These are opposite of BCBS assumptions for the developed nations. This implies that large corporate loans have the same systematic risk in times of economy distress. Our analyses suggest that it is imprudent to assume a decreasing relationship between average asset correlation and default probability in measuring portfolio credit risk. In light of this empirical evidence, we encourage the Basel Committee to revisit the use of this relationship in bank capital requirement.

Introduction

1.1 Introduction

A healthy banking system is a fundamental condition for financial stability. When assessing the riskiness of the banking system, little is known about their credit risk. The reason is that these banks usually resolve financial distress within their own organizations, which means probability of default (PD), is not observable from the outside.

PDs have proven to be powerful input factors for the internal risk controlling and risk adjusted pricing, so in recent years, banks have started using rating systems for estimating PDs based on Basel guidelines. As an example, assume a rating model for the corporate sector where the input factors are balance sheet data and the output is one of six to eight possible rating grades. According to the Basel Capital Accord, banks are suppose to estimate a PD for each rating grade and the PD should be a forecast of the default rate for the following year. Obviously the future PD rate of each rating grade depends on the several factors like the present macroeconomic situation, e.g. a downturn today will increase the PD rate of tomorrow, or factors like capital, market, return, default correlation, relationship with the banks, soft information, intensities of individual obligors, timeliness, stability, accounting information, non-accounting information, loan conditions, asset correlation etc.

In modeling a portfolio credit risk, we find quite a number of these factors to have improved the predictability of default. However, Asset correlation and Probability of default are critical drivers in modeling a portfolio credit risk. Basel II Accord assumes that the average *asset correlation decreases with probability of default* and is a *better predictor of probability of default*. In this study, we for the first time, examine the empirical validity of this assumption in the loan portfolio of public sector banks in the Indian context.

Credit risk plays a dominant role in the bank's total risk. It is a risk where borrower is either not willing or, may not be able to repay its contractual obligations/debt. These default risks are highly significant for the following reasons: first, the margins are tight as such it affects the profitability; second, as banks have high leverage, any risk of default realization leads to de-capitalization. This can severely affect the reputation of a bank and even failure of the bank. The regulators are concerned and thus, make the bank to provide for its regulatory capital for any risk based reverse events, by setting conditions and limits to calculate risks. The banks, within these conditions and limits, choose its own default models to accurately calculate the risk, which allows them to have less capital reserve.

In the bank, credit risk scoring models are used to estimate the creditworthiness of a loan and thereafter, it is grouped into risk bands. These are more commonly known as credit ratings. There are two important factors: accuracy of inputs and validation of the rating model. On 20 March, 2007, Reserve Bank of India final guidelines was released with respect to Basel II norms implementation and on 02 May, 2012, Reserve Bank of India final guidelines was released with respect to Basel III norms implementation with respect to credit risk ratings and more broadly risk.

Following this, throughout this period, the banks in India have been designing internal credit risk rating models known as the Internal rating based approach (IRB) for the estimation of capital for credit risk. These internal models help to measure the counter party risk and price the counter party risk. It helps the intrinsic risk to be systemized in the loan process, and therefore, the regulatory capital of the banks. One of the key inputs in this internal model is the borrower's (individual/portfolio) prediction of probability of default (PD) which also helps the bank to identify the non-defaulting borrowers from the defaulting borrowers, to take lending decisions, and to take pricing strategies. This PD is expressed in percentage. It is also known as the frequency of loan default. Currently, banks in India use Standardized IRB method for calculating the regulatory capital for risk coverage which is guided by the RBI. In future, as per Basel guidelines banks are expected to use their own advanced IRB method for calculating the regulatory capital which will require prediction of PD. This is where we are motivated to find out whether PD with asset correlation is a better predictor than PD without asset correlation.

Quantification of credit risk is known as the expected loss which is a product of PD (the frequency), loss given default (the cost to the bank) and exposure at default (amount of loan). This measure is used to provide for the regulatory capital. Thus, accurate prediction of PD frequency will help in providing for accurate regulatory capital. Thus, PD helps to measure the credit risk by measuring the default frequency.

These credit losses can be more severe, if the borrowers are two or more, and these default simultaneously. This type of loss is known as the portfolio risk and happens when there are two or more assets in a bank portfolio. The higher is the simultaneous default more commonly known as default correlation; the greater is the portfolio risk concentration. Whereas, the lower is the default correlation; the greater is the portfolio diversification. Therefore, the dynamics of correlation of default is also a critical key to manage a portfolio. This quantification of the concentration risk is a critical part of credit risk management (CRM) by banks.

These concentration risks can either be an exposure concentration risk which is related to credit exposure to a single sector, or industry, or borrower or group; or it can be a correlation risk based on common factors leading to simultaneous default between sectors, or industries, or borrowers. Thus, correlation or joint dependence describes the magnitude to which loans default simultaneously. It can happen between borrowers with same production inputs; or with same geographical market. It can also happen when one borrower default triggers other borrower's default. It can also happen when many borrowers default simultaneously due to industry specific economic distress or recession. Thus, the banks internal model calculates the probability of default (PD) and the default correlation between borrowers in a portfolio. This allows them to reduce the risks by taking in the diversification effects.

Default models make use of ratings (credit risk score/risk bands) and asset value models/intrinsic models (also known as structural/reduced form approaches/models) which are popular for estimating PD and default correlation for large corporate/portfolio/sovereign. We use the structural model for our study.

Based on balance conception of creditworthiness/solvency, structural models estimate default risk by using *market* information. This model was introduced by Merton (1974) with the principal that- pay off to shareholders is similar to pay off a European call option. This is like having a call option by the owners on the company's value of asset, and the strike price is the loan outstanding. It uses Black and Scholes' model (1973) option pricing to estimate PD. When the value of the company's asset becomes less than the loan value, default occurs. As such, the option is exercised at maturity, only if, the value of the company's asset is more than the value of the loan. So that, the surplus is shared after the loan has been paid off. First passage time, Copula and Factor based approaches are some approaches to estimate correlation in structural models. Among these, factor based model is the most popular one, which is simply a Merton (1974) option theory extension. In this model, the asset value consists of two components: the systematic and the idiosyncratic and both have a zero mean with normal distribution. The advantage of structural model is that it is more flexible, it uses more observations, it can be updated along with asset value of a firm and it can be generalized. CreditMetrics (Gupton et al., 1997) is the most popular factor based structural model. Our study is limited to calculating the PD using implied asset correlations for credit ratings in a portfolio of loan in public sector banks derived from implied asset correlations for industries which is in turn is derived from default correlation based on market information using the CreditMetrics (CM) default model.

In our study, the implied asset correlation are derived from default correlation by use of the asymptotic single risk factor (ASRF) model (Gordy and Heitfield, 2002) from actual one year bank transition matrix covering a period of 2000-2010, using 5461 firm corporate account migrations, in a loan portfolio from three public sector banks in India. This method takes into account the importance of default correlation as well as asset correlation.

According to the CM model, a firm's rating is determined by a band of asset value known as asset thresholds. These are based on estimation of asset value probabilities, and are also known as Conditional PD (BCBS, 2005). At the end of a time horizon, the new ratings are determined by the fall or increase in asset value to a certain asset value band. It is assumed that these percentage changes of assets are distributed normally. These are called asset returns. The asset thresholds are calculated using the transition matrix for each rating category. Thereafter, using inverse normal distribution, asset returns scenarios are generated. These scenarios are then mapped with asset threshold based band of credit scenarios.

According to the Creditmetrics approach, default of a loan occurs when the loan book value is more than the market value of borrower's assets. It proxies equity correlation for asset correlation, and being a factor based model; it is based on Merton (1974) work. The Conditional PDs are calculated from Average PDs, which are the original PDs based on historical transition matrix. This mapping procedure is an extension of the Merton (1974) 'single asset model to credit portfolios' or simply, the option model. Here, obligor's default, if they fail to meet their commitments at a fixed time horizon, because their value of the asset is less than the amount of loan due for payment. Merton uses this asset value as a normally distributed random variable which can change with time.

BCBS (2005) mentions that banks regulatory credit risk model should be portfolio invariant. This means regulatory capital required should be calculated only from the risk of an individual loan and must not depend on the portfolio. It means capital allocation is risk rating based. As such, portfolio invariance will strongly influence a portfolio structure because of concentration risk. An ASRF (asymptotic single risk factor) model (Gordy and Heitfield, 2002) is derived from ordinary portfolio model but, still it is a portfolio invariance model, because of law of large numbers. The theory behind this is that in a portfolio there will be many small exposures. And, the individual idiosyncratic risks associated in this case will cancel out; left out will be the systematic risk that affects the whole portfolio materially on portfolio losses. In this model, all the system wide risks called the systematic risks which affect all loans to a certain degree are modeled on a single risk like industry.

BCBS (2005) sites the above reason for its preferable choice of ASRF. This model uses average PD under normal business environment. The systematic risk factor is considered same for all the loans in a portfolio; hence it has the same value and reflects the state of economy through asset correlation. This has two meanings. First, asset correlation means how much the borrower's asset value is dependent on the state economy. Second, it means how much one borrower's asset value is depended on another borrower's asset value. Hence, the importance of PD with asset correlation. This model does not treat specifically any concentration and/or diversification aspects of the actual portfolio. Further, Vasicek (2002) extends the Merton Model to ASRF model. This means extending the systematic and idiosyncratic risk factors to conditional PD or the asset value through normal distribution. We use the same method in our study.

We have used the ASRF model as an input to the CM model. In the CM model, the appropriate asset threshold is estimated by reversing the Merton Model to the PDs of transition matrix called the average PDs (APD). According to Merton model, the conditional PD and the asset threshold are related to each other through normal distribution, as such, the asset thresholds are first estimated by inverse normal distribution to the APD, then from it, the conditional PD are derived by normal distribution. This is like deriving the input from the output, and then deriving the output again from the input. Similarly, the inverse normal distribution is applied to the already determined asset correlation matrix (market based) through ASRF model to determine a new conditional asset threshold. Our study uses Cholesky factorization for this mapping. This is then used as an input for original risk transformation through mapping.

Thus, the reasons for using CM model are: First, it uses a portfolio approach which is widely used by banks. Second, CM estimates correlation among asset values to derive PD with correlation by using correlation between industries/sectors. To achieve this, CM first defines the industry/sector/activity of each obligor/firm, and then estimates the weight of this industry/sector/activity for each obligor. It then combines these weights with industry/sector/activity correlations to calculate the asset correlation of an obligor/firm. Third, we are able to apply the ASRF model into the CM approach. The drawback of this model is that it is considered not enough precise due to its simple assumption for considering the same PD for borrowers in the same rating category.

Further, modeling portfolio risk in credit portfolios is neither analytically nor practically easy. Fundamental differences between credit risks and equity price risks make equity portfolio theory problematic when applied to credit portfolios. There are two problems. The first problem is that equity returns are relatively symmetric and are well approximated by normal or Gaussian distributions. Thus, the two statistical measures – mean (average) and standard deviation of portfolio value – are sufficient to help us understand market risk and quantify percentile levels for equity portfolios. In contrast, credit returns are highly skewed and fat-tailed

Thus, we need more than just the mean and standard deviation to fully understand a credit portfolio's distribution. This long downside tail of the distribution of credit returns is caused by defaults. Credit returns are characterized by a fairly large likelihood of earning, a relatively small profit through net interest earnings, coupled with a relatively small chance of losing a fairly large amount of investment. Across a large portfolio, there is likely to be a blend of these two forces creating the smooth but skewed distribution shape above.

The second problem is the difficulty of modeling correlations. For equities, the correlations can be directly estimated by observing high-frequency liquid market prices. For credit quality, the lack of data makes it difficult to estimate any type of credit correlation directly from history. Potential remedies include either: (i) assuming that credit correlations are uniform across the portfolio, or (ii) proposing a model to capture credit quality correlations that has more readily estimated parameters. In summary, measuring risk across a credit portfolio is as necessary as it is difficult. CreditMetrics methodology addresses much of this difficulty.

CreditMetrics (Gupton et al. 1997) uses a transition matrix which has been estimated from the historical data. This matrix shows the borrower's probability of credit rate drifting. And, in a bank, mapping the borrower's drift to default is the basic integral part of credit risk management. The lending decisions warrant the predictability of probability of a default of a borrower on a continuous basis besides checking on the discriminatory power of IRB. This also helps in better pricing and cushioning regulatory capital. Thus, correct predictions of probability of default are very important as incorrect predictions will ultimately reduce shareholder value. This is the aim of this study: to predict the accurate frequency probability of default using the important factors: default correlation and from it, the derived asset correlation in a portfolio by using single systematic risk factor across industries for credit risk ratings and compare the results with the estimated Probability of default without any asset correlation.

Therefore, modeling default and credit quality correlation require understanding the risk profile of a credit portfolio of a bank and estimating the default dependence. In this matter, an asset correlation factor (BCBS, 2005, 2006) is specifically included in calculation of credit risk capital by banks in its internal ratings approach, and thus, the importance of PD with asset correlation.

In a portfolio, the relationship between default and asset correlation and the initial credit quality has been well documented but without any conclusion. Zhou (1997) finds that asset correlations are higher than the default correlations. High credit firms have low default correlations and low PD. Das et al. (2002) findings are against Zhou (1997) findings. In a study, De Servigny and Renault (2002) and Bluhm and Overbeck (2003) find that there is a correlation between credit events which is supported by historical default rates and with increase in credit risk, default correlation also increases. However, it is against the findings of Das et al (2002). Lopez (2004) finding are in line with the BCBS (2001c, 2005, 2006) saying that 'asset correlation is a decreasing function of PD'. Bandyopadhyay et al. (2007), in the Indian context, findings on corporate bonds are inconclusive and almost against BCBS (2006). This is supported by Lee et.al (2009) who suggests that asset correlation is almost an increasing function of PD. From the above, we can observe that, there is no consensus on the relationship between the implied asset correlation, and the PD.

Our study evidences that probability of default improves with asset correlation. We also find that asset correlation is an increasing function of probability of default. High rating firms have low correlation than low rating firms. This implies that large corporate loans have the same systematic risk in times of economy distress. In light of this evidence, we encourage the Basel Committee to revisit the use of this relationship in bank capital requirement.

1.2 Outline

This study consists of three parts. The first part gives the introduction and provides the overall background information, the motivation and the aim of the study. The second part reviews the relevant literature on PD, discusses the data, the method and the result of this study under various sections. Finally, the third and the last part discuss the implications, the limitations and conclusions for this study.

Predicting the Probability of Default

2.1 Literature Review

On 20 March, 2007, Reserve Bank of India final guidelines was released with respect to Basel II norms implementation and on 02 May, 2012, Reserve Bank of India final guidelines was released with respect to Basel III norms implementation with respect to credit risk ratings and more broadly risk capitalization.

In response to this, banks in India have been designing internal credit risk rating models known as the Internal rating based approach (IRB) for the estimation of capital for credit risk. One of the key inputs in this internal model is the borrower's (individual/portfolio) prediction of probability of default (PD) which also helps the bank to identify the non-defaulting borrowers from the defaulting borrowers, to take lending decisions, and to take pricing strategies. Currently, banks in India use Standardized IRB method for calculating the regulatory capital for risk coverage which is guided by the RBI. In future, as per Basel guidelines banks are expected to use their own advanced IRB method for calculating the regulatory capital which will require prediction of PD. In some cases, a few private banks have started with this model. But Basel II also requires that in the independent advance stage of IRB, banks are supposed to predict the PD with the asset correlation. The importance of this correlation has already been discussed in the introduction. We give further literature on this area in this section. In this matter, an asset correlation factor (BCBS, 2005, 2006) is specifically included in calculation of credit risk capital by banks in its internal ratings approach guidelines, and thus, the importance of PD with asset correlation. This is where we are motivated to find out whether PD with asset correlation is a better predictor than PD without asset correlation.

In the bank, credit risk scoring models are used to estimate the creditworthiness of a loan and thereafter, it is grouped into risk bands. There are two important factors: accuracy of inputs and validation of the model. At the same time, Regulatory capital provisions needs quantification of credit risk known as expected loss. It is depended on three factors: the exact frequency with respect to PD; the cost to the bank (Loss given default); and amount of the loan (exposure at default). While source of the data on the latter two are totally internal system generated, the former that is estimating the PD, needs market information, and has element of uncertainty. Usually, this quantification process therefore has two dimensions: estimate the PD and calculate the loss given default (Yu et al. 2001).

PDs are assessed by default models and credit scoring models. Default models are applied to large loans and portfolios. It models the default process while credit scoring are statistical in nature, uses risk indicators for instances of default. Default models are merged with correlation models to measure portfolio risks (multiple exposures). These are then known as the portfolio risk models. When a default model is created from historical rating migration frequencies (called transition matrices), then these models are known as ratings migration models. CreditMetrics calculates PD with transition matrix for a portfolio. Hence, it is a portfolio risk-rating migration model. It is used to model the portfolio's market value over a time horizon. This is done by using the asset value model, also known as the structural model. The structural model assesses credit risk of a corporate debt. This model is also known as the Merton model as Merton (1970, 1974) anticipated this model.

Merton (1974) structural model estimates risk and PD of corporate bonds. The model assumptions are: 'value of the asset follow geometric Brownian motion with a constant volatility; capital structure has only common equity and deep discount debt; the market is perfect; stock holders receive no dividends; and debt holders are paid at the maturity of debt.' The model follows the principal that pay off to shareholders is similar to pay off a European call option. This is like having a call option by the owners on the company's value of asset, and the strike price is the loan outstanding. It uses Black and Scholes' model (1973) option pricing to estimate PD. When the value of the company's asset becomes less than the loan value, default occurs. This option is exercised at maturity, only if the value of the company's asset is more than the value of the loan. So that, the surplus is shared after the loan has been paid off. The advantage of this model is that no accounting information is needed as well as anticipation of financial status in the future. Black and Cox (1976) made improvement to the model by relaxing the assumption that firm's default only at maturity by introducing the concept of default barrier.

Correlation describes the direction and degree of a relationship between two asset values/sectors/industries in a loan portfolio. The two extreme cases may be that: there exists no correlation between firms' different unique and specific factors and a firm's credit quality is depended upon individual specific events. Then a change in the market has no effects. On the other hand, if there is a perfect correlation, then many firms would default together. However, looking at the historical data of default, we observe the existence of correlation between firms, but neither of the cases is completely correct. The portfolio approach tries to optimally decrease this risk by reducing the concentration risk through a good diversification of portfolio components; and by reducing the incremental risk which measures the sensibility to a change in portfolio component. Correlation indicates the portfolio components relation with the economic events and with each other. Thus, it helps to achieve optimal asset allocation.

The factor based model assumes default, as an event, happen as soon as the asset value crosses a critical threshold which is based on borrower's loan value. The default variable has two components: systematic and idiosyncratic. These are uncorrelated, have zero mean and are distributed normally. According to this model, a firm's rating is determined by a band of asset value known as asset thresholds. At the end of a time horizon, the new ratings are determined by the fall or increase in asset value to a certain asset value band. Default is the worst case. CreditMetrics uses asset value to estimate rating migration and default probability by defining thresholds for each rating and comparing these with the asset value of a firm for change in asset quality at the end of a time horizon. To calculate the PD, it is assumed that percent changes in assets known as asset returns are normally distributed and the asset thresholds are calculated using the transition matrix for each rating category. Thereafter, using normal distribution, asset returns scenarios are generated. These scenarios are then mapped with asset threshold based band of credit scenarios.

Gersbach and Lipponer (2000), Servigny and Renault (2002) and Gordy and Heitfield (2002) uses asset correlation to calculate default correlation using factor model, whereas, Crouhy et al (2000), Zhang et al (2008), Bandyopadhyay and Ganguly (2011) uses default correlation to calculate asset correlation using factor model. All the study is based on Lucas (1995) and Nagpal and Bahar (2001) calculation of joint probabilities. Bandyopadhyay et al. (2007) use Bluhm et al. (2003) model to calculate the implied asset correlation from default correlation for corporate bonds. The method is estimating bivariate transition probabilities from historical data. Bandyopadhyay et al. (2007) in the Indian context also find that default correlation (DC) have business cycle effect and vary across industries and ratings. They find high DC between firm of same ratings which is systematic risk impact and high DC between firms of same industry (specific impact). But do not find any smooth monotonic relationship between PD and asset correlation. This is in contrast to BCBS (2006). It is also supported by Lee et.al (2009). This is against the BSBS (2005) and implies the evidence of the dependence of firms on macroeconomic factors.

In structural model, equity correlation proxies for asset correlation, but its relation with default correlation is very noisy (Servigny and Renault, 2002). Lopez (2004) finds that there is relatively high asset correlation between high rated large size firms since they are more affected by common macroeconomic conditions. Whereas it is low because of firm specific problems for low rated large size firms. BCBS (2006) highlights this relationship by saying that as PD decreases, asset correlation also decreases.

Further, BCBS (2005), evidences that default by corporate loans of bank in a portfolio are strongly related to the state economy and interactions between such loans are high. At the same time, it finds that defaults in a retail portfolio have low correlation reflecting the fact that it has more idiosyncratic risk with less dependence on economic conditions as well as the inter-linkage between such loans are poor.

However, BCBS (2005, 2006) based on the finding and analyses of data from the G10 countries have mentioned the following dependencies: First, based on empirical and intuition, they mention that ‘Asset correlations decrease with increase in PD.’ The intuition states high PD will have high idiosyncratic risk and less systematic risk. Second, again based on empirical and intuition, they mention that ‘asset correlations increase with firm size.’ Large firms will have more systematic risk and less idiosyncratic risk as they will be more dependent on economy and vice-versa. Third, asset correlation has a lower and an upper limit- 12% for 100% PD and 24% for 0% PD. Clearly, the first and the second findings are contradictory for large corporate accounts. Further, the findings are based on developed economy. Finally, Basel II Accord assumes that the average *asset correlation* is a *better predictor of probability of default*. In the given scenario, our study examines the validity of this assumption in the loan portfolio of public sector banks in the Indian context as well as provides evidence to the relationship between probability of default and implied asset correlation for loan portfolio in Indian public sector banks.

2.2 Data and Method

We have, used primary data collected from three public sector banks, for estimating the one year transition matrix. A total of 5461 loan accounts transitions from all the industries covering a period 2000-2010 were used for the purpose of which 553 accounts are default transitions. Thus, the data used has two groups: non-defaulted firms and defaulted firms. The transition matrix is given in table 2.1below along with the one year transition probabilities percentage matrix for the period 2000-2010 in table 2.2.

Ratings	a1	a2	a3	a4	a5	a6	a7	a8	total
a1	388	33	66	21	0	1	1	1	511
a2	27	459	201	53	23	2	1	3	769
a3	29	148	1574	257	49	9	10	17	2093
a4	24	47	231	749	54	20	5	17	1147
a5	2	2	21	59	141	16	2	30	273
a6	0	2	7	13	19	54	7	30	132
a7	0	0	7	6	4	7	21	48	93
a8	0	0	1	1	2	15	17	407	443
total	470	691	2108	1159	292	124	64	553	5461

Table 2.1 One year transition matrix for the period 2000-2010

Ratings	a1	a2	a3	a4	a5	a6	a7	a8
a1	75.92955	6.457926	12.91585	4.109589	0	0.195695	0.195695	0.195695
a2	3.511053	59.68791	26.13784	6.892068	2.990897	0.260078	0.130039	0.390117
a3	1.385571	7.07119	75.20306	12.27903	2.341137	0.430005	0.477783	0.812231

a4	2.092415	4.097646	20.13949	65.30078	4.707934	1.743679	0.43592	1.482127
a5	0.732601	0.732601	7.692308	21.61172	51.64835	5.860806	0.732601	10.98901
a6	0	1.515152	5.30303	9.848485	14.39394	40.90909	5.30303	22.72727
a7	0	0	7.526882	6.451613	4.301075	7.526882	22.58065	51.6129
a8	0	0	0.225734	0.225734	0.451467	3.386005	3.837472	91.87359

Table 2.2 One year Transition probabilities % matrix for the period 2000-2010

With the data, we first segregated the portfolio into 11 industries based on firm's activity. The industry is selected according to bank's internal policy, where the portfolio is divided into 19 broad industry categories. We reduced this to 11 industries based on NIC codes. We merged rubber, plastic, petroleum, nuclear and coal with chemical; beverage and tobacco with food; Gems and jewelry, leather, wood and construction with diversified; glass and cement with metal. We have assigned only one industry/sector to each firm/obligor before combing them irrespective of their representing one or more sectors. We also treat the firms being active in India only irrespective of their representing one or more countries. We choose this to avoid complexity of calculations and are one limitation of this study. It is also based on the data limitation.

We have, industry wise, for default correlation and then implied asset correlation, used the data from annual ratings of CRISIL's long term bonds, for estimating the one year default correlation covering a period 1995-2010, for 2162 bond issues, and 91 defaults. This qualifies our method as we are now using the market information. Default in this case has been defined as short fall in terms of payment by even a single paisa, or delayed payment by even a single day. We give below the industry classification and the default probability of the above data for the period 1995- 2010 in table 2.3.

Industry name	Type	Firms	Defaults	Probability of Default
Auto	Manufacture of transport equipments, parts, commercial vehicle, car, ancillaries, bicycle, two-three wheeler, aircraft, ship. Boats, railway	174	5	2.873563
Chemical	Organic, inorganic chemicals manufacturing, chemical product, dyes, paints, rubber, plastic, photo goods, coal products, petroleum products	503	20	3.976143
Diverse	Diversified industries	70	5	7.142857
Food	Food products, dairy products, tea, sugar, vegetable oil, coffee, fats, bakery, beverages, tobacco, breweries and related products	81	6	7.407407
Machine	Machinery, equipment manufacturing, electrical, electronic, engineering, computers, wires, cables, fire machinery, industrial machinery-food, textile, construction	250	19	7.509881

Metal	Basic metal, alloys, iron, steel, Ferro alloys, copper, steel tubes, aluminum, transmission towers, cement, mica, glass, ceramic, refractory and other non metal	288	19	6.597222
Other	Optical goods, trading all types	46	0	0
Paper	Paper, paper products, newsprint, printing, publishing, allied activities	53	3	5.660377
Power	Power, electricity, roads, telecommunications	66	0	0
Service	Hotel, insurance, banking, other financial services	481	6	1.247401
Textile	Cotton, wool, silk, man-made fiber, jute and other textile manufacturing	147	8	5.442177

Table 2.3 Industry classification and default probability for the period 1995-2010

Next, we use the ASRF model, which is a portfolio invariance model for estimating the asset correlations by inculcating the systematic and idiosyncratic weights into the model for each obligor. This model is then extended into the Creditmetrics methodology which is a factor based approach of structural model for estimating PD with correlation.

We use the CM methodology to estimate the PD with asset correlation of a portfolio in this study as it uses correlation among industries/sectors to calculate correlation among obligors in a portfolio to estimate the PD. We first calculate the joint PD, PD and default correlations of these industries on approach developed by Nagpal and Bahar (2001) and Lucas (1995) and used by Bandyopadhyay et al. (2007) and De Servigney and Renault (2002). Then we use asymptotic single risk factor (ASRF) methodology, and derive the asset correlation among industries from the JPD, PD and default correlation through iteration. This methodology has been used by Gordy and Heitfield (2002), Bluhm et al. (2003), Bandyopadhyay and Ganguly (2011) and recommended by BCBS (2005). Then the weight of each industry is calculated from the indices; and from this, the idiosyncratic weights are calculated. Thereafter, the estimated asset correlation and both the weight are combined to arrive at the asset correlation among each obligor in a portfolio.

As already mentioned earlier that we have assigned only one industry/sector to each firm/obligor before combing them irrespective of their representing one or more sectors. And, that we also treat the firms being active in India only irrespective of their representing one or more countries. We choose this to avoid complexity of calculations and are one limitation of this study. This may have an effect on the weights of indices for each obligor which when calculated were found to be small, this in turn, increases the idiosyncratic weights of each obligor. Overall, this affects the magnitude of asset correlations as well as default correlations. Also, a drawback of CM model is that it is considered not enough precise due to its simple assumption for considering the same PD for borrowers in the same rating category.

CM uses historical data based transition matrix to calculate PD. These are, and then converted into asset value based PD using asset value threshold bands for each class of credit rating for a borrower. We then generate random scenarios (20000 in this study) using cholesky factorization for each obligor asset value using the implied asset correlation among all the individual obligors.

These are then mapped to credit ratings for each obligor using the asset value thresholds bands. This is then averaged out to calculate the PD with correlation.

Transition matrices are prepared from historical rating transitions. The bank uses the traditional technique known as the cohort approach with simple average in preparing the transition matrices. These have no effect of sequencing a transition or timing and borrowers hold a rating at the beginning of a time horizon. The time horizon in our study is one year. The bank uses simple average method for the following reasons: number of accounts is very large; rating of the borrowers is a round the year process as such it is difficult to have a standard cutoff time for the one year time horizon; PD depends more on macro-economic factors than number of accounts. A rating transition happening between 1st January and 31st December of a year is treated as a transition for that period from its earlier rating. We used 5461 banks internal rating transitions covering a period 2000-2010. The transition matrix is already given in Table 2.2. The figure in the matrix is known as one year transition probabilities of a given rating in a portfolio. We use the following ratings: AAA/A1; AA/A2; A/A3; BBB/A4; BB/A5; B/A6; CCC/A7; and D/A8.
To calculate the default correlation, we use the eleven industries as mentioned above, which are Auto; Chemical; Diverse; Food; Machine: Metal; Other; Paper: Infrastructure; Service; Textile. The industry name, type, number of firms and number of defaults are already given in Table 2.3. To estimate the default correlation, mortality rate analysis is done for each one year cohorts of firms moving to default, where default is defined as short fall in terms of payment by even a single paisa, or delayed payment by even a single day. This way our sample of data for default correlation is represented by 100% market. From these cohorts, for each industry, year wise PD is calculated first and then the weighted average. The weights are calculated by dividing total accounts in an industry in a given year by total accounts in the sample period for the industry. It is given as:

$$PD_i = \sum_{t=1}^{n} w_i^t \frac{T_{i,D}^t}{N_i^t} \quad (2.1)$$

Where PD_i is the average one year PD for *i*th industry; $\frac{T_{i,D}^t}{N_i^t}$ is the EDF or PD, and is the number of accounts transiting to default (D) for an industry by total accounts in that industry; and w_i^t is the relative weight, which is defined above and is given by:

$$w_i^t = \frac{N_i^t}{\sum_{s=1}^{n} N_i^s} \quad (2.2)$$

Second step is counting the joint default frequency by comparing defaulting accounts pairs at the end of a time horizon with total number of pairs at start of a period (time horizon) starting within same industry and then with other industries. For example: Auto-Auto; Auto-Chemical, etc. Thus, joint default probability (JDP) with-in an industry for one year will be: $(T_{i,D})^2 / (N_i)^2$ and the average JDP is given by:

$$JDP_{i,i} = \sum_{t=1}^{n} w_i^t ((T_{i,D}^t)^2 / (N_i^t)^2) \quad (2.3)$$

Where w_i^t is as defined in (2.2).

Similarly, it is $(T_{i,D}T_{j,D})/(N_iN_j)$ for between industries and the average JDP is given by:

$$JDP_{i,j} = \sum_{t=1}^{n} w_{ij}^t ((T_{i,D}^t T_{j,D}^t)/(N_i^t N_j^t)) \quad (2.4)$$

Where the weight $w_{ij}^t = \frac{(N_i^t + N_j^t)}{\sum_{s=1}^{n}(N_i^s + N_j^s)}$ (2.5)

Finally, the default correlation (DC) is calculated for one year as:

$$\rho_{i,j}^{D,D} = \frac{JDP_{i,j} - PD_iPD_j}{\sqrt{PD_i(1-PD_i)PD_j(1-PD_j)}} \quad (2.6)$$

This method is based on Lucas (1995), later on developed by Nagpal and Bahar (2001). This method assumes that PD and JPD are constant over time which is a limitation of this methodology. The DC is given in Table 2.4. The one year JPD with correlation between the industries in % is given in Table 2.5. The PD for the respective industries is already given in table 2.3.

	Auto	Chem	Diverse	Food	Machine	MNM	OtherM	Paper	Power	Service	Textile
Auto	2.96	0.00	-0.61	-0.61	1.02	1.32	0	0	0	1.32	0.24
Chem		2.81	3.12	1.96	2.61	2.40	0	0	0	1.03	1.62
Diverse			21.26	10.70	12.37	4.40	0	0	0	5.89	-0.36
Food				10.44	5.89	2.81	0	0	0	3.72	1.96
Machine					9.77	5.00	0	0	0	5.45	-0.02
MNM						4.38	0	0	0	2.96	0.63
OtherM							0	0	0	0	0
Paper								0	0	0	0
Power									0	0	0
Service										4.28	0.35
Textile											5.37

Table 2.4 One year default correlation in % across industries for the period 1995-2010

Third step is to derive the implied asset correlation (AC) from the DC so that the results can be compared to BCBS (2006) specification with respect to asset correlation. We take the ASRF approach. This model computes the AC using the Merton model where default is assumed to have occurred, as soon as the asset value A_i crosses the default threshold. This realized return are normalized and are sum of the weight of the common systematic risk factor and borrower specific idiosyncratic risk factor. These are uncorrelated to each other.

	Auto	Chem	Diverse	Food	Machine	MNM	OtherM	Paper	Power	Service	Textile
Auto	0.17	0.11	0.18	0.19	0.26	0.24	0	0.18	0	0.06	0.17
Chem		0.27	0.44	0.39	0.43	0.38	0	0	0	0.07	0.29
Diverse			1.92	1.25	1.38	0.75	0	0	0	0.26	0.37
Food				1.26	0.96	0.67	0	0	0	0.20	0.52
Machine					1.24	0.82	0	0	0	0.25	0.41
MNM						0.70	0	0	0	0.16	0.39
OtherM							0	0	0	0	0
Paper								0.99	0	0	0
Power									0	0	0
Service										0.07	0.08
Textile											0.57

Table 2.5 One year joint default probability with correlation between industries in % for the period 1995-2010

We assume $A_i \sim N(0,1)$; default is triggered when default threshold say Z_i crosses A_i, thus $PD_i = \Pr[A_i \le Z_i] \Rightarrow Z_i = N^{-1}(PD_i)$; Z_i is therefore, a function of *i*th industry rating. Therefore, JDP is: $JDP_{ij} = \Pr[A_i \le Z_i, A_j \le Z_j] = N(Z_i, Z_j, \rho_a)$ (2.7)

Where $N(.)$ is the 'cumulative bivariate standard normal distribution'; ρ_a is the asset correlation; and Z_i, Z_j are the default threshold. We calculate ρ_a from (3.7) above. We know JDP_{ij} from table 2.5, PD_i & PD_j from table 2.3, DC from table 2.4 and estimate ρ_a (AC) for the industry through iterations (given in Table 2.6) using the BIVNORF function which gives the $N(.)$, and, where:

$JDP_{ij} = BIVAR(norm\sin v(PD_i), norm\sin v(PD_j), \rho_a)$ (2.8)

And, DC is given by (2.4) above. Gordy and Heitfield (2002) used this function to measure JDP.

Fourth step is using the CreditMetrics (Gupton et al. 1997) method of generalized asset correlation for all the 11 (eleven) industries and all the ratings by preparing an 11*7 by 11*7 asset correlation matrix. We follow the process given in the CreditMetrics document. We use the AC from Table 2.6, the beta of the industry as the indices part which accounts for the movement of the equity *(w)* (the systematic component), and idiosyncratic component derived from this systematic component as: $\sqrt{1-w^2}$. To create a matrix which has all these three components, first we construct an *m+n by m+n* matrix M, where the upper left of the matrix is the AC from Table 2.6 (11 by 11, or m by m) and the lower right (77 by 77, or n by n representing 11 industries portfolio and 7 ratings up to CCC/A7) matrix reflecting each industry/rating portfolio having a one correlation for itself and zero correlation with other industry/rating portfolio. The upper right and the lower matrix part are all zero showing no correlation between AC and idiosyncratic component. Next, a weight matrix K is created where each column stands for an industry/rating and each row first stands for the weights on indices and then weights on the idiosyncratic component respectively for each column (given in table 2.6). The AC matrix for the portfolio (77*77) is given by $K' * M * K$.

Industry	Auto	Chem	Diverse	Food	Machine	MNM	OtherM	Paper	Power	Service	Textile	IW*	IDW**
Auto	14.86	0.03	-3.00	-3.00	4.55	6.00	0	0.03	0	9.50	1.23	0.1485	0.98
Chem	0.03	12.20	11.63	7.60	9.80	9.38	0	0	0	7.05	6.90	0.1218	0.99
Diverse	-3.00	11.63	47.85	28.55	31.90	13.90	0	0	0	25.95	-1.43	0.4785	0.87
Food	-3.00	7.60	28.55	27.82	17.25	9.23	0	0	0	18.15	6.95	0.278	0.96
Machine	4.55	9.80	31.90	17.25	26.25	15.35	0	0	0	24.35	-0.06	0.265	0.96
MNM	6.00	9.38	13.90	9.23	15.35	14.07	0	0	0	15.38	2.45	0.141	0.99
OtherM	0	0	0	0	0	0	0	0	0	0	0	0.001	0.9999
Paper	0.03	0	0	0	0	0	0	0	0	0	0	0.003	0.999
Power	0	0	0	0	0	0	0	0	0	0	0	0.01	0.99
Service	9.5	7.05	25.95	18.15	24.35	15.38	0	0	0	26.80	2.40	271	0.962
Textile	1.23	6.9	-1.43	6.95	-0.06	2.45	0	0	0	2.40	18.20	0.185	0.987

*Indices weight; **Idiosyncratic weight

Table 2.6 Asset correlation between industries

In the fifth step, CreditMetrics uses an asset value model which is an extension of the Merton (1974) option model to link the change in credit rating with asset value of a firm. It assumes that a firm's asset determines the debt paying ability and if the asset value falls below a threshold level, it defaults. It uses this principle to create a series of asset threshold level for each credit rating. Thus, to give a new credit rating at the end of a period, we have to model the asset value change in percent known as the asset returns (*R*) with the asset threshold level. It is assumed that the R is normally distributed. Thus, we have an asset threshold for each rating: $Z_{def}, Z_{CCC}, Z_B, Z_{BB}, Z_{BBB}, Z_A, Z_{AA}, Z_{AAA}$. So that, for example, when, $R < Z_{def}$, there is a default. And, if it is $Z_{def} < R < Z_{CCC}$ the asset is given a rating of CCC. We use transition matrix probabilities to calculate these asset threshold probabilities and are called 'probability according to asset value model'. It is calculated as:

$$\Pr_{default} = \Phi(Z_{def} / \sigma)$$

$$\Pr_{CCC} = \Phi(Z_{CCC} / \sigma) - \Phi(Z_{def} / \sigma)$$

And so on, where $Z_{def} = \Phi^{-1}\sigma$; Φ is the cumulative normal distribution and σ is the volatility of the asset return. For each rating, the asset return threshold is calculated and is given in table 2.7 below.

rating	Threshold	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC
A1 AAA								
A2 AA	*ZAA*	-0.70404	1.810482	2.201348	2.035026	2.440868		
A3 A	*ZA*	-0.93023	-0.33713	1.374988	1.539013	2.179372	2.166107	
A4 BBB	*ZBBB*	-1.675	-1.24464	-0.98057	0.633218	1.331118	1.48947	1.437635
A5 BB	*ZBB*	-2.51981	-1.77789	-1.74363	-1.38063	0.502402	0.967422	1.081286
A6 B	*ZB*	-2.51981	-2.41803	-2.11535	-1.79137	-0.9314	0.494133	0.904762
A7 CCC	*ZCCC*	-2.65941	-2.56213	-2.2292	-2.07098	-1.18902	-0.58194	0.649324
A8 DEFAULT	*ZDef*	-2.88502	-2.66051	-2.40337	-2.17483	-1.22711	-0.74786	0.040441
		-10	-10	-10	-10	-10	-10	-10

Table 2.7 Asset Returns threshold for each rating

To estimate the migration of two assets jointly, the CreditMetrics assumes they are normally distributed and there is correlation ρ between the two assets. It is computed as:

$$\Pr\{Z_B < R < Z_{BB}, Z'_{BBB} < R' < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r, r'; \Sigma)(dr')dr \quad (2.9)$$

Where r, r' are the values of the two assets returns; $f(r, r'; \Sigma)$ is the density function; Σ is the covariance matrix given by: $\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma' \\ \rho\sigma\sigma' & \sigma'^2 \end{pmatrix}$ for two firms with ratings of BB and A; and with probability to keep the same rating. But, it is not possible to calculate joint probabilities with correlation for each pair of asset returns in a portfolio. CreditMetrics provide for an alternative scenario generation option. We generate future credit ratings scenarios for our portfolio of 77 obligors based on 11 industry and 7 ratings according to normal distribution and then map the asset returns with the credit ratings. For this, we use Cholesky factorization method and generated 20,000 correlated normally distributed scenarios for each obligor. We first use the correlation matrix to decompose it using cholesky factorization. Then we generate 20000 independent standard normal random variates. These are then converted into 20000 scenarios for standardized asset returns.

The method is as follows: to generate a vector, X, of dependent normal variates with means μ and covariance matrix V from cholesky decomposition. Let C be the cholesky factor of V (it means V= C* C^T) and let Z be a row vector of independent standard normal random variates. Then Y= $\mu + C * Z^T$ and X= Y^T . This X are the new 20000 generated asset returns which is then mapped to the credit rating scenarios using the band of asset threshold, in the process generating 20000 new credit ratings for each obligor. Finally, we add up all the individual rating scenarios for a given obligor and then take out the percentage of each rating for an obligor. This way we generated PD with asset correlation for each of the 77 industry/rating. We then take an average of the generated PD with asset correlation for each rating by merging the industries into one obligor for each rating. This is given in Table 2.8. Table 2.9 shows the average JPD for ratings; table 2.10 shows asset correlation on rating basis; and table 2.11 shows default correlation on ratings basis respectively. This is done by repeating the earlier method discussed with respect to industry based correlations calculations. Table 2.12 gives the percentage difference between PD with correlation and PD without correlation.

Rating	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC	A8 DEF
A1 AAA	69.76273	5.402727	12.71227	6.887273	0	0.645	0.898182	3.680455
A2 AA	9.615455	50.53864	22.28136	7.532273	4.830909	0.679091	0.393182	4.118182
A3 A	6.309545	8.924545	61.795	12.50136	3.400909	0.796818	1.003182	5.256364
A4 BBB	7.561364	5.194545	18.36045	53.49318	5.372727	2.553182	0.75	6.700455
A5 BB	4.894091	1.502273	9.417727	18.905	41.32818	5.345909	0.692727	17.90091
A6 B	0	6.577273	6.865455	9.480909	11.91455	32.45864	4.428182	28.26182
A7 CCC	0	0	14.19273	6.297273	3.866364	6.362727	17.79409	51.47318

Table 2.8 Transition matrix for PD with asset correlation for period 2000-2010

AJPD rating %							
	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC
A1 AAA	0.0010476	0	0.001525	0.000577	0.036516	0.006559	0.102503
A2 AA		0.004744	0.003638	0.011879	0.056165	0.222277	0.309454
A3 A			0.007308	0.016164	0.11318	0.203493	0.480182
A4 BBB				0.052042	0.187277	0.602298	1.040409
A5 BB					2.022526	3.633976	8.201878
A6 B						14.49056	24.04879
A7 CCC							42.36364

Table 2.9 One year joint default probability with correlation between ratings in % for the period 2000-2010

	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC
A1 AAA	6	0	15	50	12	9	2
A2 AA		10	10	14	8	30	22
A3 A			12.1	10	5.7	3	6.5
A4 BBB				15	4	20	19
A5 BB					24.2	18.5	34.5
A6 B						77	99
A7 CCC							83.55

Table 2.10 one year Asset correlation in % between ratings for the period 2000-2010

	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC
A1 AAA	0.15	0	0.86	8.6	1.22	0.98	0.22
A2 AA		0.49	0.63	1.19	1.01	4.96	3.07
A3 A			1.04	0.99	0.92	0.55	1.27
A4 BBB				2.03	0.78	5.13	4.56
A5 BB					10.35	8.67	16.19
A6 B						53.14	52.5
A7 CCC							62.96

Table 2.11 One year default correlation in % across ratings for the period 2000-2010

	A1 AAA	A2 AA	A3 A	A4 BBB	A5 BB	A6 B	A7 CCC	A8 DEF
A1 AAA	-8.121769	-16.3396	-1.57619	67.5903	0	229.595	358.9709	1780.712
A2 AA	173.86239	-15.3285	-14.7544	9.289014	61.5204	161.1105	202.3568	955.6273
A3 A	355.37513	26.20996	-17.8291	1.810716	45.2674	85.30449	109.966	547.1511
A4 BBB	261.37017	26.76901	-8.83359	-18.0819	14.12071	46.42498	72.05	352.0836
A5 BB	568.04341	105.0602	22.43045	-12.5243	-19.9816	-8.78543	-5.44273	62.89827
A6 B	0	334.1	29.46286	-3.73231	-17.2253	-20.6567	-16.4971	24.352
A7 CCC	0	0	88.56052	-2.39227	-10.107	-15.4666	-21.1976	-0.27071

Table 2.12 Percentage difference between PD with correlation and PD without correlation

2.3 Results

We have, used primary data collected from three public sector banks, for estimating the one year transition matrix. The transition matrix for the one year probability of migration and default is given in table 2.2. As expected, we find the diagonal of the matrix with high probability load as the borrower are expected to maintain their current ratings. This is true for all the ratings except A7/CCC. The ratings are mentioned in a column and their rating transition probabilities given in respective rows. If there is a transition then the maximum transition will be from these diagonals, as such, the second highest probabilities will be adjacent to these diagonals. This is also evidenced in our matrix. This rule is known as the row monotonicity property of transition matrix (Violi, 2004). We find this rule breaks down with increase in transition horizon as with our exception of A7/CCC. This is because the default rates now become prominent and in a portfolio the default is the ultimate absorbing state in a credit risk. This is the reason for jump in probability of default from A4: 1.5 %, to A5:10.99%, to A6:22.7%, to A7:51.6%. This justifies why bank does not sanction a loan with an initial A5 internal rating. It is also the reason why with fall in ratings, stability of ratings declines diagonally. This is true for our matrix with an exception of A3. This can be justified with the reason that all new accounts are usually given this rating at the time of a loan sanction and are the reason why it is higher than A2 (A2: 60% and A3: 75%).

Another exception we observe with the matrix is comparatively a low probability of 76% at A1:A1 diagonal. This is again because of the bank practice, a bank IRB will give A1 ratings only to the best, as such the A1 ratings will have more chances of slipping down. Finally, we find that the PDs increase with fall in rating in a monotonic manner from A1 to A7. These are: 0.2%; 0.4%; 0.8%; 1.5%; 10.99%; 22.7%; and 51.6% respectively. These PDs are calculated without any correlation effects.

To estimate the PDs with asset correlation, we first calculated the default correlation of the industry. These are given in Table 2.3 along with industry name, type, number of sample data, and number of defaults. Table 2.4 shows the one year default correlation (DC) in % across industries; table 2.5 shows the one year JPD with correlation between industries in %; and table 2.6 shows the implied asset correlation calculated from the default correlation using the ASRF method. We used the method described in the earlier section to calculate the JPD and DC across the industries.

As expected the JPD is highest along the diagonal with one exception for the machine industry which has a high JPD with diverse industry showing their dependence on diverse industry. High JPD along the diagonal signifies dependence on the same systematic risk and growth prospects, hence, more chance of simultaneous default. Thus, banks should avoid the diagonal concentration risk. As the correlation among the off-diagonal is low, banks can reduce their losses by exploiting these diversifications. The highest JPD and the highest DC is between the diverse industries (1.92% and 21.26%), followed by between Diverse and machine industries (1.38% and 12.37%). This is used with the DC to calculate the asset correlation among the industries as mentioned earlier.

We find that asset correlation (AC) between industries is higher than the default correlations (DC) and have the same signs. This confirms the finding of Zhou (1997). The AC follows the same rankings as the DC rankings between different industries. We used this AC between the industries to calculate the AC between each obligor/industry/rating wise. For this we also calculated the indices weight and idiosyncratic weight (Table 2.6) using BSE stock indices. We found the indices weight to be very small. As mentioned earlier this is due to two reasons: we treated one industry for each obligor, only India as the activity center. This is one limitation of our study. Incidentally, CreditMetrics has provision for all the activities in all the activity centers to be included for indices weight.

This asset correlation table is then used to generate 20000 scenarios for each obligor/industry/rating and using the cholesky factorization method is matched with the asset value thresholds calculated by normal distribution from the average PDs (given in table 2.7) for each obligor/industry/ratings. These are then mapped with the asset value return bands for each obligor to generate 20000 new ratings with asset correlations. These 20000 new ratings are then averaged into PDs in percentage for all the ratings for each obligor/industry/rating. We find that rating and industry wise, in A7/ccc rating, the PD is almost same for all the industries between 51.04% and 52.01%. This reduces to 19.8% for diverse and 4.9% for food in A1/AAA ratings. One implication of this is to break down the diverse industry into more different categories to understand the concentration risk.

Finally, these ratings are clubbed together to generate the transition matrix on PDs with asset correlation. The results are given in table 2.8. The estimated probabilities of default with asset correlation for each rating are as follows: A1:3.7%; A2:4.2%; A3:5.3%; A4:6.7; A5: 17.9%; A6:28.3%; A7:51.5%. From the table 2.8, we observe that the estimated PDs with correlation are higher when compared to PDs as given in table 2.2 for normal PDs. This evidences that predicting probability of default with asset correlation is a better predictor**.** The percentage increase is given in Table 2.12. The percentage increase is an increasing function for the PDs in a monotonic manner as the ratings go up but the transition probabilities across the diagonal have decreased in absolute manner as expected which has been absorbed by the default state. Thus, Banks should include asset correlation with PD in IRB.

To understand this relation better between the ratings instead of the industries, we calculated first the average JPD between the ratings (given in Table 2.9), then the default correlation across the ratings (Table 2.11) and the asset correlation (2.10) across the ratings as mention in the earlier section. We find that the asset correlation and the default correlation increases as the rating go down in a smooth monotonic manner. This is against the BSBS (2005) condition one which was the findings of G10 countries, but confirms the condition two that large corporate firms are more dependent on the economy and thus have more systematic risk. Thus, G10 (developed nations) findings in their nations do not apply in our case is empirically evidenced. We find that high ratings have low default correlation and low asset correlation. The findings have major implications.

First, PD with asset correlation is a better predictor. Second, the by-results evidences that the default correlation confirms the dependence of firms on macroeconomic factors. This should be taken into account while managing credit portfolios. Third, the by-results evidences that the asset correlation is not limited between 12% and 24% per BCBS (2005) guideline but is between 0 to almost 1 as well as there is direct relationship between PDs and asset correlation which is contradictory to Basel guideline showing that the large corporates assets are more dependent on the economy and thus, have high systematic risk embedded. Thus, IRB approach should include this for calculation of risk weights. These have serious implication for regulatory capital requirements of the banks. Fourth, the by-results implies that an arbitrage opportunities develops for the develop nations.

As per BCBS (2005), G10 nations have high asset correlation for high ratings and low correlations for low ratings which is just the opposite of our findings. This gives the G10 nations to invest in our nations in firms with high ratings less asset correlation than their nations in comparison, but with the option to withdraw the investment as soon as the economy shows a distress sign, since, now these invested firms ratings fall as well as correlation risk increases, and no arbitrage opportunity exists for the developed nations invest here and hence, they withdraw the funds.

Conclusions

When assessing the riskiness of the banking system, little is known about their credit risk. The reason is that these banks usually resolve financial distress within their own organizations, which means probability of default (PD), is not observable from the outside. PDs have proven to be powerful input factors for the internal risk controlling and risk adjusted pricing, so in recent years, banks have started using rating systems for estimating PDs based on Basel guidelines. According to the Basel Capital Accord, banks are suppose to estimate a PD for each rating grade and the PD should be a forecast of the default rate for the following year. Obviously the future PD rate of each rating grade depends on the several factors like the present macroeconomic situation, capital, market, return, default correlation, relationship with the banks, soft information, intensities of individual obligors, timeliness, stability, accounting information, non-accounting information, loan conditions, asset correlation etc.

In modeling a portfolio credit risk, we find quite a number of these factors to have improved the predictability of default. However, Asset correlation and Probability of default are critical drivers in modeling a portfolio credit risk. Basel II Accord assumes that the average asset correlation decreases with probability of default and is a better predictor of probability of default. In this study, we examined the empirical validity of this assumption in the loan portfolio of three public sector banks in the Indian context.

The objective of this study is to estimate the probability of default (PD) with asset correlation in a loan portfolio in the Indian context using a default model. The CreditMetrics (CM) model with ASRF model method is used to first estimate the asset correlation from default correlation between industries and then using it to predict the PD with asset correlation of a portfolio, it is then used to estimate the asset correlation and default correlation between ratings. We used the ASRF method (Gordy and Heitfield, 2002) to calculate the asset correlation among industries from default correlation among industries by estimating the joint PDs and using the BIVNORF function of Gordy and Heitfield (2002). It is also the preferred method according to BCBS (2005). The advantage of ASRF method is it is a portfolio invariance method and yet in can be extended to the CM model which is portfolio model. CM uses this asset correlation between industries to estimate the PD with asset correlation between different asset value bands.

CM does this by first identifying the activity of each borrower in the industry and geographical area based, then it uses the systematic and idiosyncratic weight of each borrower according to industries and area of operation with the asset correlation between industries estimated earlier using the ASRF method to estimate the borrower's asset correlation matrix for each borrower in the portfolio. This is then mapped with the portfolio asset value thresholds which are calculated using the Merton (1974) asset value model from the average PD. This average PD is used from the transition matrix estimated using historical data on rating transitions. Several scenarios for each borrower are generated using normal distribution and then using cholesky factorization, these are mapped to arrive at an average for each borrower at different asset threshold values. This is then averaged out to generate the transition matrix for PD with asset correlation value. From this value, we again used the ASRF method to generate the default correlation and asset correlation of the portfolio on rating basis by estimating the joint PDs. We used this to study the relationship among default correlation, asset correlation and PD with and without asset correlation.

Our data consists of 5461 actual private large corporate loans from three public sector banks with 553 defaults covering a period from 2000-2010. Thus, the data used has two groups: non-defaulted firms and defaulted firms. We divided the whole portfolio into 77 categories one for each industry/rating. This is assigning one industry to each borrower according to its main activity based on banks activity code and then merging them according to its asset value threshold. We also assumed that the entire firm's activity is in India even though they may have activities outside India. The reason for this is the complexities of arriving the systematic weights based on multiple activities/areas and limitation of data related to this. As a result some of the firms may have ended up with small systematic weights which may have affected the asset correlation between the borrowers.

Contrary to this effect, the results of our study evidenced that there is asset correlation between borrowers, which increases with increase in PD, and also, increases with fall in ratings in a monotonic manner and as such, *PD with asset correlation is a better predictor*. We found the same results with respect to default correlation with PDs. Our study also evidenced that asset correlation are larger than default correlation and follow the same sign. The study also evidences that high rating firms are almost uncorrelated compared to low rating firms. The study also evidences that joint PDs increase the PDs. This shows that large corporate have high systematic risk in an industry. As the literature is inconclusive with these relations, the empirical results of our study have major implications in the Indian context. First, the default correlation confirms the dependence of firms on macroeconomic factors. This should be taken into account while managing credit portfolios. Effective diversification in a portfolio is possible only through dynamic calculations of asset and default correlations between each possible pair of borrowers.

Second, the asset correlation is not limited between 12% and 24% but is between 0 to almost 1 as well as there is direct relationship between PDs and asset correlation which is contradictory to Basel II showing in India that the large corporates assets are more dependent on the economy and thus, have high systematic risk embedded. Thus, Indian bank's IRB approach should include this for calculation of risk weights. These have serious implication for regulatory capital requirements of the banks as estimating the PD (frequency) is one major dimension in estimating the loss and is only dimension with uncertainty. The Third an arbitrage opportunities develops for the develop nations. As per BCBS (2005), G10 nations have high asset correlation for high ratings and low correlations for low ratings which is just the opposite of our findings. This gives the G10 nations an option to invest in our nations in firms with high ratings less asset correlation than their nations in comparison, with the option to withdraw the investment as soon as the economy shows a distress sign, since, now these invested firms ratings fall as well as correlation risk increases, and no arbitrage opportunity exists for the developed nations to invest here and hence, withdraw the funds.

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Abstract:

We use the asymptotic single risk factor model, which is a portfolio invariant model and preferred by BCBS with the factor based structural Credit Metrics portfolio default model to empirically estimate the Probability of default with asset correlation of a loan portfolio based on primary data from Public Sector Banks and compared the results with the estimated Probability of default without any asset correlation. We have used actual bank loan rating transition data for the period 2000-2010. Our study evidences that probability of default improves with asset correlation. We also find that asset correlation is an increasing function of probability of default. High rating firms have low correlation than low rating firms. These are opposite of BCBS assumptions for the developed nations. This implies that large corporate loans have the same systematic risk in times of economy distress. Our analyses suggest that it is imprudent to assume a decreasing relationship between average asset correlation and default probability in measuring portfolio credit risk. In light of this empirical evidence, we encourage the Basel Committee to revisit the use of this relationship in bank capital requirement.

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