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EXISTENCE OF CAPITAL MARKET EQUILIBRIUM
IN THE PRESENCE OF HERDING
AND FEEDBACK TRADING

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EXISTENCE OF CAPITAL MARKET EQUILIBRIUM IN THE PRESENCE OF HERDING AND FEEDBACK TRADING

This paper attempts to establish the existence of equilibrium, in an asset market inhabited by two representative investors with different risk aversions. In order to capture heterogeneity in information and wealth, the paper segments the investor population into two: (i) Individual investors and (ii) Institutional investors. Based on prior literature, the present study posits that Institutional investors demonstrate rational intentional herding and positive feedback trading (buy when the markets rise and sell when it falls) and individual investors demonstrate negative feedback trading (vice versa). In other words, when the markets are (monotonically) increasing, institutional investors, expecting the trend to continue would buy more, thus demonstrating decreasing absolute risk aversion. Similarly, when the market is (monotonically) decreasing he will try to stem his loss as soon as possible, demonstrating increasing absolute risk aversion. Such an investment behavior is captured in a power utility function. Further, negative feedback trading by individual investors implies that when market is (monotonically) increasing individual investors, expecting the trend to reverse, would sell. Thus demonstrating increasing absolute risk aversion. And when the markets are (monotonically) decreasing, they would hold on to their investments expecting better times to come, thus depicting decreasing absolute risk aversion. Such investment behavior is captured by a quadratic utility function. Given their wealth and investment behavior, the two investor groups would trade with each other such that the market clears. To the best of our knowledge this is the first paper that proposes a asset pricing model that not only allows for behavioural biases but also for heterogeneous agents who are affected differently by these biases. This paper establishes the bounds for the absolute risk aversion function and the shadow rate of interest at which the two investor groups will lend money to each other to enable trading and market clearing. For reasonable endowments and presumed behavioural biases as implied by the chosen utility function, a numerical example at the end of this paper shows that the market clearing interest rate (at which the investors would lend to and borrow from each other) occurs between 15.5% and 28.05%.

Key Words: Heterogenous Agents, Herding, Feed back trading, shadow rate of interest.

JEL Classification: G12, C62, D53

INTRODUCTION

Since Sharpe (1965), Lintner (1966) and Mossin (1967), there have been a lot of attempts to arrive at more accurate asset pricing models. While the initial Asset Pricing Models (APMs) assumed that all investors in the market have the same expected returns and these are constant across time. The Stochastic Discount Factor (SDF) approach to asset pricing tries to relax this assumption. The initial SDF[2] based APMs assumed that all market participants have equal access to information, equal wealth and thus similar assessment of risk and expected returns. Given this assumption, the said SDF based APMs allowed for time variation in expected

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[2] SDF is the discounted ratio of marginal utility (of a representative investor) at time *t+1* to that at time *t*

returns. The later SDF based APMs allowed for heterogeneity in information, wealth, risk assessment and hence expected returns, such as: (i) APMs with agents who are constrained for resources (Constantinides and Duffie, 1996) and (ii) APMs with agents who are constrained for information (Admati, 1985).

Though the heterogeneous agent APMs are closer to how the markets work in reality, these models assume that investors are rational under all circumstances. Recently, this assumption has come under heavy criticism from the behaviorists who argue that there exists non-standardized investor behaviour which is driven by irrationality. In other words, the investor may not act rationally under all circumstances. The present paper, while attempting to stay within the Arrow-Debreu framework, tries to incorporate two of the cognitive biases, identified in existing literature: (i) Herding and (ii) Feedback Trading in the investment behaviour of market participants. Based on recent literature, it has been conjectured that as compared to institutional investors, individuals are less informed and more vulnerable to psychological biases (Kaniel *et. al.*, 2008, Barber and Odean, 2008). Accordingly, the paper segments investor population into two: (i) Individual investors and (ii) Institutional investors and posits that while institutional investors demonstrate rational intentional herding and positive feedback trading (Nofsinger and Sias, 1999), individual investors demonstrate negative feedback trading (Shefrin and Statman, 1985). Such investment behavior, demonstrated by institutional and individual investors, is captured by a specifying a power (institutional investor) and quadratic (individual investor) utility function. When these two investor types trade competitively by borrowing and lending money to each other, the allocation of wealth fluctuates randomly between them. The challenge here is to find the shadow rate of interest (at which the two investors borrow and lend) such that a equilibrium asset price exists. To our knowledge this is the first paper attempting to build in rational herding and feedback trading into rational asset pricing models. For reasonable endowments and presumed behavioural biases as implied by the chosen utility function, a numerical example at the end of this paper shows that the market clearing interest rate (at which the investors would lend to and borrow from each other) occurs between 15.5% and 28.05%.

BACKGROUND TO THE STUDY

As stated earlier, in order to capture heterogeneity in information and resources, the present paper segments the investor population into two: (i) Individual investors and (ii) Institutional investors. These two categories of investors differ in their investment behavior primarily on the following premises:

(i) Agent principal relation: Institutions manage funds provided to them by investors. Since the people who manage the funds are not the people who own them, the likelihood of agency problem is very high. One proposed solution is to link the fund managers compensation to its performance. However, in doing so, agents may at times take decisions that may not be in the best interest of owners (Guerrieri and Kondor, 2009). In the case of individual investor since the owner and manager of funds is the same individual, there is no agency problem.

(ii) Legal Environment: As compared to individual investors, institutional investors face a relatively stricter regulatory environment with regard to capital requirements, redemptions, investment strategies etc.

(iii) Liquidity and transaction cost: Institutional investors churn their portfolio much more frequently as compared to individuals (Schwartz and Shapiro, 1992). Hence, they would be more sensitive to transaction costs, especially if the asset is not liquid.

The response of individual and institutional investors to new information would differ based on their access to information and their vulnerability to psychological biases. The paper allows for (rational intentional) herding and feedback trading.

(i) Herding: It is witnessed when a group of investors trade in the same way over a period of time. Herding can be purely accidental because of arrival of correlated information to independently acting investors. Such herding is called spurious herding (Bhikchandani and Sharma, 2001). Herding can also be intentional evolving from interactive observation of actions and payoffs of other market participants. Two alternative explanation of such intentional herding are:

a. Investor Psychology: Intentional herding may be driven by psychological factors such as a tendency among investors to conform their investment choice to others. Thus, investors may take similar investment choices because they converse with each other and want to conform (Shiller, 1995) or because they observe each others' investment choices (Bhikchandani et. al., 1992).

b. Rationality: In the presence of asymmetry in information availability and processing capability, some investors may feel that they are better off (rational) by imitating another investor (Devenov and Welch, 1996). Such behavior is more prominent in cases where performance appraisal of the investment professional is relative to how his industry peers have performed. While, the less able professional may try to imitate the better professional, it has been seen that even the able investment professional would choose to follow a investment decision of the majority of his peers even if it is suboptimal to preserve his reputation in the job market (Scharfstein and Stein, 1990). This idea is premised on assumption that damage to

reputation in the job market, due to a potential failure, outweighs the expected benefits from a potentially successful investment decision (Graham, 1999).

In the present study we consider only rational intentional herding.

(ii) Feedback trading: Feedback trading occurs due to a high correlation between current investment decision and past return performance of the asset. If the investor trades in the same direction as the past return performance, then he is called a positive feedback trader, while if he trades in the opposite direction then he is called a negative feedback (contrarian) trader (De Bondt and Thaler, 1985; Bowman and Iverson, 1998; Jegadeesh and Titman, 2001; Kang *et. al.*, 2002; Antonious *et. al.*, 2005). Fundamentally, feedback trading behavior is based on the premise that information (Private/Public) takes time to get impounded into stock prices either because it is private information or because the quality of the information is suspect. Hence, initial prices (when new information is initially absorbed in stock price) tend to produce directional patterns (trends) for certain periods of time. Feedback trading behavior is an outcome of investor psychology. For example, positive feedback trading may exist because of the following behavioral biases: (a) The investor gives more weightage to more recent information as compared to older information (*Representation heuristic*); (b) the investor may be conservative in updating his beliefs in response to new information. Resultantly, new information would only gradually get reflected in stock prices (*Conservatism Bias*), thus producing directional patterns in trade (Barberis *et. al.* 1998); (c) Investor may trade aggressively after having executed a profitable trade. In other words, he may exhibit overconfidence in his investment choice following a successful trade (*Overconfidence Bias*) (Odean, 1998). Further, negative feedback trading may be caused by the reluctance of people to sell stocks whose performance in the recent past has been poor and to sell stocks whose performance has been good (*Disposition Effect*) (Shefrin and Statman, 1985).

Accordingly, the present study posits the following:

(i) Rational intentional herding is relevant only in the case of institutional investors since they tend to have more symmetric information as compared to individuals and also because, their labour market reputation is linked to the performance of their investment.

(ii) Though, feedback trading is witnessed among both the investor groups (institutional and individual), prior literature states that while institutional investors exhibit positive feedback trading (Nofsinger and Sias, 1999), individual investors demonstrate negative feedback trading (Shefrin and Statman, 1985).

THE MODEL

The capital market of our model economy consists of two types of investors: (i) Institutional investors and (ii) Individual investors. To map their investment behavior two separate utility functions are specified.

Institutional Investor: If the institutional investor demonstrates herding and positive feedback trading then, when the market outlook (expected payoff) is positive and the asset prices are expected to go up, because he expects past trends to continue, he would invest more money in the market. Thus, demonstrating a decrease in his absolute risk aversion during an uptrend in asset prices. Similarly, when the when market outlook is gloomy, he will try to stem his loss as soon as possible, thus demonstrating an increase in absolute risk aversion during a downtrend in the market. Such negative relation of absolute risk aversion coefficient with expected payoff is captured by a power utility function $U_1(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$ (where, $C > 0$ and $0 < \ < 1$).

Individual Investor: If the individual investor demonstrates negative feedback trading then, when the market outlook is positive and asset prices are increasing, he tends to book profits pretty soon and exit, thus demonstrating an increase in his absolute risk aversion during an uptrend in the market. Similarly, when the market outlook is gloomy, he tends to hold on to the asset in the hope that it will be profitable in the future. By doing so, he demonstrates a reduction in his absolute risk aversion during a downtrend in the market. Such positive relation of absolute risk aversion coefficient with expected payoff is captured in a quadratic utility function, $U_2(c^*) = ac^* - \dfrac{b}{2}c^{*2}$, where $b < \dfrac{a}{c^*}$.

The two investors consume a single good and have access to two investment opportunities:

(i) They can invest in stocks in such a way that their investment is linearly homogenous to capital with the expected returns following a Gaussian distribution with parameters   and   . They can borrow/ lend capital with each other at a rate   , which varies endogenously over time.

Notations

Let w^* be the wealth of the individual investor; w be the wealth of institutional investor. Say, s is the aggregate wealth in an economy *i.e.* $s = w^* + w$. Let x^* and x be the proportion of wealth of the individual and institutional investor respectively that is invested in risky assets. And let c^* and c be the consumption rates of individual and institutional investors respectively.

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Assuming that the dynamics of aggregate wealth in an economy are governed by an *Ito* process, they can be specified as:

$$ds = (\alpha s - c - c^*)dt + \sigma s dz$$

Welfare Optimum and Existence of Equilibrium

Given that this study posits a power and quadratic utility function for institutional and individual investors respectively, to attain (pareto) optimal welfare, a weighted average of the two investors' utility is to be maximized. Say and (*1-*) are the weights associated with the utility functions of institutional and individual investors respectively and is the discount factor, the equation for welfare optima can be stated as:

$$\max_{c,c^*} E_0 \int_0^\infty e^{-\rho t}[(1-\lambda)(ac^* - \frac{b}{2}c^{*2}) + \lambda(\frac{c^{1-\gamma}}{1-\gamma})]dt \tag{1}$$

where, $0 < \lambda < 1$, $0 < \gamma < 1$ and $\rho > 0$

subject to $ds = (\alpha s - c - c^*)dt + \sigma s dz$ given S_0 and $S_t \geq 0$.

Necessary Condition

A necessary condition for maximizing the previous equation is that there exists a function *v(s)*, the undiscounted bellman function such that the Hamilton Jacobi Bellman equation (hereafter HJB), the first order conditions and the "transversality conditions" at infinity are satisfied. The HJB for equation (1) can be written as:

$$0 \equiv (1-\lambda)(ac^* - \frac{b}{2}c^{*2}) + \lambda(\frac{c^{1-\gamma}}{1-\gamma}) - \rho v + v'(\alpha s - c - c^*) + \frac{1}{2}v''\sigma^2 s^2 \tag{2}$$

at optimum:

$$v'(s) = \lambda c^{-\gamma} = (1-\lambda)(a - bc^*)$$

$$\therefore c = (\frac{\lambda}{v'(s)})^{\frac{1}{\gamma}} \text{ and } c^* = (\frac{a}{b} - \frac{v'(s)}{b(1-\lambda)}) \tag{3}$$

and transversality condition is

$$\lim_{T \to \infty} E_0 e^{-\rho T} v'(s_T)s_T = 0 \tag{4}$$

Substituting the optimal values of *c* and c^* in the HJB and differentiating with respect to *s* gives:

$$0 = -\rho v'(s) + v'(s)\alpha + v''(s)[\alpha s - \left(\frac{\lambda}{v'(s)}\right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{v'(s)}{b(1-\lambda)}\right)] + \frac{1}{2}v'''(s)\gamma^2 s^2 + v''(s)\sigma^2 s$$

(5)

Defining p as the marginal utility of wealth, $p_t \equiv p(s_t) \equiv v'(s_t)$, equation (5) can be written as:

$$0 = -\rho p + p'[\alpha s - \left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{\rho}{b(1-\lambda)}\right)] + \frac{1}{2}p''\sigma^2 s^2 + p\alpha + p'\sigma^2 s$$

(6)

Defining the shadow rate of interest for the welfare problem as: $r_t \equiv r(s_t) = p\alpha + p'\sigma^2 s$

equation (6) could be written as:

$$(\rho - r)p = p'[\alpha s - \left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)}\right)] + \frac{1}{2}p''\sigma^2 s^2$$

(7)

The right hand side of equation (7), is the conditionally expected change in marginal utility of wealth p. By a simple application of Ito's lemma, the change in p can be expressed as:

$$\left. \begin{aligned} dp(s) &= p'ds + \frac{1}{2}p''(ds)^2 \\ \text{where,} \, ds &= [\alpha s - \left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)}\right)]dt + \sigma s dz \\ \text{and} \, \lim_{T\to\infty} E_0 \, e^{-\rho T} p_T s_T &= 0 \end{aligned} \right\}$$

(8)

$$\left. \begin{aligned} \therefore dp(s) &= p'[\alpha s - \left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)}\right)]dt + p'\sigma s dz + \frac{1}{2}p''\sigma^2 s^2 dt \\ &= (\rho - r)pdt - \left(\frac{\alpha - r}{\sigma}\right)pdz_t \end{aligned} \right\}$$

(9)

For the optimal solution to exist, the quantum of present wealth in the economy should be equal to the sum of the discounted future wealth (Detailed proof given in Appendix 1).

$$p_0 s_0 = E_0 \int_0^\infty e^{-\rho t} p_t \left[\left(\frac{\lambda}{p_t} \right)^{1/\gamma} - \left(\frac{a}{b} - \frac{p_t}{b(1-\lambda)} \right) \right] dt \qquad (11)$$

where the dynamics of the total wealth in the economy will be governed by

$$ds = \left[\alpha s - \left(\frac{\lambda}{p_t} \right)^{1/\gamma} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)} \right) \right] dt + \sigma s dz \qquad (12)$$

Given that the undiscounted Bellman function $v(s)$ exists and is strictly increasing, concave and twice differentiable one can define the aggregate utility of consumption in the market $u(x)$ as:

$$u(x) = \max_{c,c^*} (1-\lambda)[ac^* - \frac{b}{2} c^{*2}] + \frac{\lambda}{(1-\gamma)} (c^{1-\gamma}) \qquad (13)$$

subject to $c + c^* = x$

Based on this definition of aggregate utility in the market, the conditions necessary for an optimal solution to exist can be arrived at. In other words, the bounds for the absolute risk aversion function as well as the shadow rates of interest such that the market clears and an optimal solution exists, can be arrived at. The said bounds for the absolute risk aversion function is given as (Proof elaborated in Appendix 2)

$$1 >< -\frac{u''(x)}{u'(x)} >< [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]\gamma \qquad (14)$$

Thus, the Bellman function v(s) would also satisfy a similar property (Dumas, 1989)

Therefore, $1 >< -\dfrac{v''(s)}{v'(s)} >< [\dfrac{1}{c(x)} + \dfrac{x - c(x)}{c(x)} - \dfrac{a}{bc(x)}]\gamma$

We will use this result to arrive at the bounds in which the shadow rate of interest of lending and borrowing should move so that the market clears and the equilibrium exists.

we know $r(s) = \alpha + \dfrac{v''(s)}{v'(s)} \sigma^2 s$

By (14) we can say that

$$\alpha - \sigma^2 s > < \alpha + \frac{v''(s)}{v'(s)}\sigma^2 s > < [\frac{a}{b(x)} - \frac{1}{c(x)} - \frac{x-c(x)}{c(x)}]\gamma\sigma^2 s + \alpha$$

Or

$$\alpha - \sigma^2 s > < r(s) > < [\frac{a}{bc(x)} - \frac{1}{c(x)} - \frac{x-c(x)}{c(x)}]\gamma\sigma^2 s + \alpha$$

The two extreme values would be obtained when the individual or institutional investors hold all the wealth.

A Numerical Example

Say the aggregate consumption in an economy is 100 units and the optimal consumption for the individual investors is 30 units. The economy is inhabited by two investors (i) individual and (ii) institutional. The individual investor's investment behavior is mapped by specifying a quadratic utility function $U(c^*) = 10 * c^* - 0.04 * c^{*2}$ and the institutional investor's investment behavior is mapped by specifying a power utility function $U(c) = \frac{c^{(1-0.3)}}{(1-0.3)}$.

Assume that the return per unit of capital invested in this economy follows a Gaussian distribution with fixed mean α (=20%) and standard deviation σ (=30%). Assume also that the proportion of wealth with the individual investor is 0.5. Based on the bounds derived above, the market clearing rate of interest at which the two investors would borrow from and lend to each other will be 15.5% and 28.05%.[15.5%<(resp. >) r <(resp. >) 28.05%. This market clearing rate can also be used to calculate the equity premium.

CONCLUSION

In this paper, we have given a simple proof of existence of market equilibrium in an asset market inhabited by two types of agents: (i) Institutional Investors and (ii) Individual investors. By assuming that the institutional investor is characterized by a power utility and the individual investor is characterized by a quadratic utility, the paper tries to incorporate two irrational investment behavioral (i) Herding and (ii) Feedback trading into the asset pricing model. This paper establishes that given that the risk aversion coefficients of heterogeneous representative agents are in certain bounds we can arrive at the shadow rate of interests at which these agents would lend to /from each other such that the market clears. The equilibrium pricing so arrived at would adjust for existence of herding and feedback trading amongst market participants. Thus, in simple terms it can be said that the study tries to generalize the existence of equilibrium price even when investment behavior is irrational.

However, some comments are in order. First of all, the choice of quadratic utility constrains the utility function to be valid only till $b<(a/c^*)$, but we feel that it is more important to capture the behavioral biases in individual investor decision making and for most practical purposes this is at best a scaling issue. Secondly, the choice of quadratic utility to represent the individual investor behavior leads to an equilibrium asset pricing partial differential equation that does not have a closed form solution. We again feel that this is the tradeoff that we make to capture the behavioural bias in individual investor's decision making process. Finally, there is again a lot of heterogeneity among individual investors based on whether they are "high net worth" individuals or otherwise. There is also a lot of heterogeneity in the case of institutional investors depending on their size and skill. This paper does not deal with this issue. However a solution to this problem can be arrived at in a similar fashion (three or four agent problems) nevertheless a bit more complicated.

Appendix 1: Proof of a necessary condition for existence of Welfare optima

Lemma: A necessary condition for the optimal solution (while maximizing the weighted average of the two investors' utility) to exist is that there exists a function p(s) (the marginal utility of wealth) defining a process such that the quantum of present wealth in the economy is equal to the sum of discounted future wealth:

$$p_0 s_0 = E_0 \int_0^\infty e^{-\rho t} p_t \left[\left(\frac{\lambda}{p_t} \right)^{1/\gamma} - \left(\frac{a}{b} - \frac{p_t}{b(1-\lambda)} \right) \right] dt$$

where, the dynamis co total wealth in the economy is governed by a stochastic process:

$$ds = \left[\alpha s - \left(\frac{\lambda}{p_t} \right)^{1/\gamma} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)} \right) \right] dt + \sigma s dz$$

Proof: Given the dynamics of total wealth *ds* and the marginal utility of wealth *dp(s)* one can define a stochastic differential equation (SDE) mapping the dynamics of the co movement of p and s i.e.

$$d(ps) = pds_t + sdp_t + ds_t dp_t$$

      (A.1.1)

Substituting *ds_t* and *dp_t* from equation 8 and 9 we get

$$d(ps) = [\rho ps - p[\left(\frac{\lambda}{p} \right)^{\frac{1}{\gamma}} - \left(\frac{a}{b} - \frac{p}{b(1-\lambda)} \right)]dt + ps[\sigma - \left(\frac{\alpha - r}{\sigma} \right)]dz$$

      (A.1.2)

Taking expectations on both sides of equation A.1.2, we get

$$E[d(ps)] = E\left(\rho ps - p[(\frac{\lambda}{p})^{\frac{1}{\gamma}} - (\frac{a}{b} - \frac{p}{b(1-\lambda)})] \right) dt$$

      (A.1.3)

Now, we have a linear Ordinary Differential Equation (ODE) and integrating this with respect to boundary condition in equation 8, gives

$$p_0 s_0 = E_0 \int_0^\infty e^{-\rho t} p_t \left[\left(\frac{\lambda}{p_t} \right)^{1/\gamma} - \left(\frac{a}{b} - \frac{p_t}{b(1-\lambda)} \right) \right] dt$$

(A.1.4)

Appendix 2: Bounds for absolute risk aversion coefficient and the shadow rate of interest.

Lemma: Assuming that the undiscounted Bellman function, if it exists, is strictly increasing , concave and twice differentiable, the bounds for Arrow-Pratt absolute risk aversion coefficient, $-\dfrac{u''(x)}{u'(x)}$, and the shadow rate of interest, $r(s)$ (Such that the market clears, is given by:

$$1 >< -\frac{u''(x)}{u'(x)} >< [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]\gamma$$

and, $\alpha - \sigma^2 s >< r(s) >< \alpha - \gamma\sigma^2 s[\dfrac{1}{c(x)} + \dfrac{x - c(x)}{c(x)} - \dfrac{a}{bc(x)}]$

Proof: We know that the aggregate utility of consumption in the market is given by:

$$u(x) = \max_{c,c^*} (1-\lambda)[ac^* - \frac{b}{2}c^{*2}] + \frac{\lambda}{(1-\gamma)}(c^{1-\gamma})$$

Let $c + c^* = x$ and substitute $c^* = x - c(x)$, where $c(x)$ is the optimal c for a given x. We now have a maximization problem similar to

$$f(x, c(x)) = (1-\lambda)[a(x - c(x)) - \frac{b}{2}(x - c(x))^2] + \frac{\lambda}{1-\gamma}[c(x)]^{1-\gamma}$$

Partial derivative of $u(x)$ with respect to the first argument, $c^*(x)$, gives:

$u'(x) = (1-\lambda)[a - b(x - c(x))]$

(A.2.1)

and $u''(x) = (1-\lambda) - b(1 - c'(x))$

(A.2.2)

Therefore, the coefficient of absolute risk aversion is given as:

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$$- \frac{u''(x)}{u'(x)} = \frac{b(1-c'(x))}{a-bx+bc(x)}$$

(A.2.3)

Further, the partial derivative of $u(x)$ with respect to the second argument, $c(x)$ gives:

$$u'(x) = (1-\lambda)(-a+b(x-c(x))+\lambda c(x)^{-\gamma}$$

(A.2.4)

At optimum, $u'(x) = 0$, Therefore we can say that $(1-\lambda)(a-b(x-c(x)) = \lambda c(x)^{-\gamma}$

And, based on equation A.2.1, we can say that $u'(x) = \lambda c(x)^{-\gamma}$

Further, $u''(x) = -\gamma\lambda c(x)^{-\gamma-1}c'(x)$

Thus, the coefficient of absolute risk aversion can be written as:

$$-\frac{u''(x)}{u'(x)} = \gamma\frac{c'(x)}{c(x)}$$

(A.2.5)

To arrive at the bounds for the coefficient of absolute risk aversion, first assume that $-\frac{u''(x)}{u'(x)} < 1$ where $a,b > 0$ & $0 \le x - c(x) \le \frac{a}{b}$

Based on equation A.2.3, we can say that $b(1-c'(x)) < a-bx+bc(x)$

Therefore, $\frac{c'(x)}{c(x)} > \frac{1}{c(x)} + \frac{x-c(x)}{c(x)} - \frac{a}{b(x)}$

Since $x > 0$ and $0 \le x - c(x) \le \frac{a}{b}$ we can say that $0 < \frac{x-c(x)}{c(x)} < \frac{a}{bc(x)}$

Therefore, $1 > \gamma\frac{c'(x)}{c(x)} > \gamma[\frac{1}{c(x)} + \frac{x-c(x)}{c(x)} - \frac{a}{bc(x)}]$

Similarly, if we assume $-\frac{u''(x)}{u'(x)} > 0$, then it can be shown that

$$1 < \gamma \frac{c'(x)}{c(x)} < \gamma [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]$$

Thus, the bounds for the absolute risk aversion coefficient of the investors is given as:

$$1 >< -\frac{u''(x)}{u'(x)} >< \gamma [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]$$

(A.2.6)

To arrive at the bounds for the shadow rate of interest, it would be reasonable to assume that the Bellman function $v(s)$ would satisfy a similar property as in equation A.2.6 (Dumas, 1989).

Therefore, $1 >< -\frac{v''(s)}{v'(s)} >< \gamma [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]$

As expressed in equation 7, the shadow rate of interest for the welfare optimum is given by:

$$r(s) = \alpha + \frac{v''(s)}{v'(s)} \sigma^2 s$$

Based on A.2.6, the bounds for the shadow rate of interest can be arrived at as:

$$\alpha - \sigma^2 s >< \alpha + \frac{v''(s)}{v'(s)} \sigma^2 s >< \alpha - \gamma \sigma^2 s [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]$$

$$\alpha - \sigma^2 s >< r(s) >< \alpha - \gamma \sigma^2 s [\frac{1}{c(x)} + \frac{x - c(x)}{c(x)} - \frac{a}{bc(x)}]$$

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Abstract:

This paper attempts to establish the existence of equilibrium, in an asset market inhabited by two representative investors with different risk aversions. In order to capture heterogeneity in information and wealth, the paper segments the investor population into two: (i) Individual investors and (ii) Institutional investors. Based on prior literature, the present study posits that Institutional investors demonstrate rational intentional herding and positive feedback trading (buy when the markets rise and sell when it falls) and individual investors demonstrate negative feedback trading (vice versa). In other words, when the markets are (monotonically) increasing, institutional investors, expecting the trend to continue would buy more, thus demonstrating decreasing absolute risk aversion. Similarly, when the market is (monotonically) decreasing he will try to stem his loss as soon as possible, demonstrating increasing absolute risk aversion. Such an investment behavior is captured in a power utility function. Further, negative feedback trading by individual investors implies that when market is (monotonically) increasing individual investors, expecting the trend to reverse, would sell. Thus demonstrating increasing absolute risk aversion. And when the markets are (monotonically) decreasing, they would hold on to their investments expecting better times to come, thus depicting decreasing absolute risk aversion. Such investment behavior is captured by a quadratic utility function. Given their wealth and investment behavior, the two investor groups would trade with each other such that the market clears. To the best of our knowledge this is the first paper that proposes a asset pricing model that not only allows for behavioural biases but also for heterogeneous agents who are affected differently by these biases. This paper establishes the bounds for the absolute risk aversion function and the shadow rate of interest at which the two investor groups will lend money to each other to enable trading and market clearing. For reasonable endowments and presumed behavioural biases as implied by the chosen utility function, a numerical example at the end of this paper shows that the market clearing interest rate (at which the investors would lend to and borrow from each other) occurs between 15.5% and 28.05%.

Key Words/Phrases: Heterogenous Agents, Herding, Feed back trading, shadow rate of interest.