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ON MODELING THE STEP FIXED-CHARGE TRANSPORTATION PROBLEM

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ON MODELING THE STEP FIXED-CHARGE TRANSPORTATION PROBLEM

Fixed-charge transportation problem (FCTP) deals with determining optimal quantities of goods to be shipped and the routes to be used to satisfy the customers' demands at minimal total cost. The total cost contains a fixed component which is incurred for every route that is part of the solution along with the variable cost that is proportional to the amount shipped. Step fixed-charge transportation problem (SFCTP) is a variant of the FCTP where the fixed costs follow a step function. Staircase cost structure is very common in the shipping industry, national postal services and couriers, and materials management. In this work, we propose a MILP model for SFCTP. After explaining the mathematical model in sufficient detail, we demonstrate its applicability on a small numerical example. Using extensive computational experiments, we conclude that the problem is a very hard problem with much "higher degree" of polynomial complexity. We also report that the number of steps in the fixed component appears to be the dominant factor that significantly affects the computational time. Though the proposed MILP model is applicable for SFCTP, with minor modifications, it can be generalized and used for other network optimization problems that warrant modeling of staircase cost structures.

Keywords: *Fixed charge; Transportation problem; Step function; MILP model; Staircase costs*

INTRODUCTION & REVIEW

Network representation is often the most convenient method for modeling many real world complex problems arising in application areas such as telecommunications, transportation, and logistics. An important problem in the network optimization literature is the classical transportation problem (TP). It is the problem of determining optimal quantities of goods to be shipped to satisfy the demands at minimal total cost. Owing to its totally uni-modular (TUM) structure, even large problem instances of TP can be solved as a simple linear programming problem (LPP) thereby saving precious computational time. As a natural extension of TP, fixed-charge transportation problem (FCTP) was proposed by Balinski [1]. FCTP accounts for both fixed and variable costs along the shipping routes. The problem is a proven NP-hard problem (Hirsch and Dantzig [2], Klose [3]) and has been well researched over the past 50 years. A wide variety of solution methodologies, both exact and heuristic, are available in the literature to solve the FCTP. Interested readers can refer to Xie and Jia [4] for a comprehensive literature review on this problem.

There exist three variants of FCTP in the literature. One of the variants proposed by Kowalski and Lev [5], christened as the step fixed-charge transportation problem (SFCTP), considers fixed costs as a step function of the quantity shipped. This results in an objective function with non-convex piecewise linear costs (also called as staircase costs). It is as shown in Figure 1. Staircase cost structure is very common in shipping industry (Baumgartner et al. [6]), pricing of national postal services and couriers (Lapierre et al. [7]) and materials management (Kameshwaran and Narahari [8]). The second variant proposed by Xie and Jia [4] considers variable costs directly proportional to the quadratic of its shipping amount and thereby becoming nonlinear. In the third and last variant, researchers (Jawahar and Balaji [9], Molla-Alizadeh-Zavardehi et al. [10], Antony Arokia Durai Raj and Rajendran [11]) consider a two stage FCTP that determines the optimal shipping schedule between supply and demand nodes through a set of intermediate nodes called distribution centers (DCs) or warehouses. In addition to the fixed cost associated with each route, the models take into consideration the fixed cost for opening potential distribution centers (DCs) with capacity constraints.

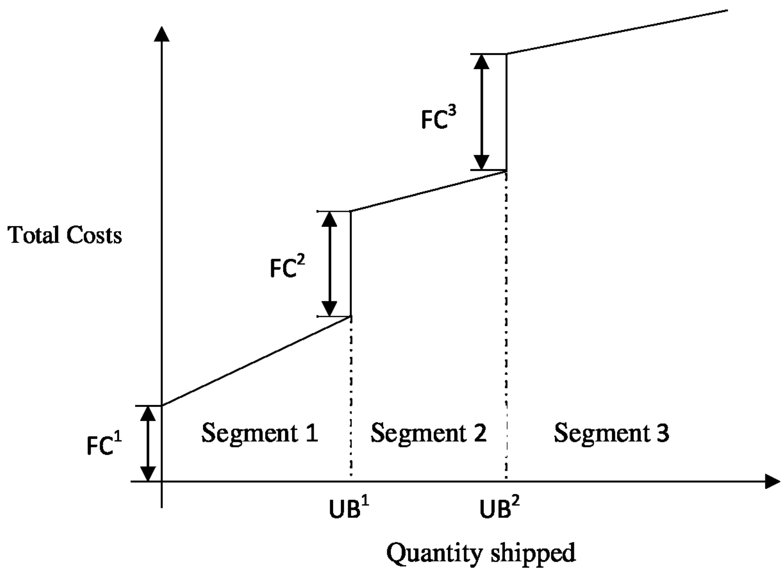

Fig. 1. Shipping cost structure of SFCTP

By restricting the scope of the current work to the first variant, we extend the work done by Kowalski and Lev [5] on SFCTP. Network flow problems with piecewise linear costs arise in many applications. Network loading problem (Magnanti et al. [12], Bienstock and Gunluk [13], Gabrel et al. [14], Gunluk [15]), facility location problem with staircase costs (Holmberg [16], Holmberg and Ling [17]), and the merge-in-transit problem (Croxton et al. [18, 19]) are some of the specific examples. In many cases, staircase cost structures are simply replaced by quite crude linear approximations or one step fixed costs, due to the difficulty of the problems [17]. On SFCTP, other than the work done by Kowalski and Lev [5], we are not aware of any published work in the literature. In their work, after describing the SFCTP, the authors propose two heuristic procedures for solving the problem. It is noteworthy to mention here that the mathematical formulation suggested by them has a non-linear objective function which makes the problem difficult to solve even for simple problem instances and also forbids from using commercially available optimization solvers such as LINDO, LINGO and IBM CPLEX because of the non-linearity.

In this work, we propose a mixed-integer linear programming (MILP) model for SFCTP based on variable disaggregation techniques [20]. The model takes into consideration fixed costs, expressed as a step function, and variable costs proportional to the quantity shipped on a given route. Despite the fact that the problem at hand is a proven NP-hard problem, we are interested in finding out the amenability of the model in real life scenarios as it is intended to be deployed at a container shipping service provider's location. Hence, we proceed to investigate a) highest problem instance of SFCTP that can be solved to optimality within 30 minutes of computational time b) quality of the lower bounds using linear programming (LP) relaxation and c) effect of different parameters on the computational time to solve a problem instance to optimality. An important contribution of this work is the MILP model that can be generalized and applied for other network optimization problems which warrant the modeling of fixed costs as a piecewise linear function.

The rest of the paper is organized as follows. Section 2 provides a detailed description of the proposed mathematical model. Next, in Section 3, we detail the characteristics of test datasets generated to evaluate the efficacy of the proposed model. Results obtained are summarized in Section 4 followed by conclusions in Section 5. The proposed mathematical model applied on a small numerical example is presented in Appendix A.

PROBLEM FORMULATION

Assumptions

1. There exist only one product to be shipped between multiple supply and demand points
2. Total supply is greater than or equal to total demand
3. Fixed costs depend only on the range in which the quantity to be shipped falls. Ranges (segments or slabs) are predetermined with specific lower and upper bounds which are non-overlapping and collectively exhaustive for the entire range of the quantity allowed.
4. The segments are the same regardless of the route chosen.
5. Variable costs do not depend on the segments. They depend only on the quantity to be shipped on any route.

Notation

m = total number of sources

n = total number of destinations

s = total number of segments/slabs/steps

S_i = units of output at source i

D_j = demand at destination j

C_{ij} = variable cost of transportation per unit between source i and destination j

FC^k_{ij} = fixed cost associated with segment k between source i and destination j

LB^k = lower bound on the number of units in segment k

UB^k = upper bound on the number of units in segment k

R^k = $UB^k - LB^k$ = range of segment k

Decision variables:

x_{ij} = quantity shipped between source i and destination j

q^k_{ij} = quantity shipped from source i to destination j that falls in segment k

Y^k_{ij} = 1 if q^k_{ij} is positive; 0 otherwise

Mathematical model:

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} FC_{ijk}Y_j \quad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le S_i \quad \forall i = 1 \ldots m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} \ge D_j \quad \forall j = 1 \ldots n \quad (3)$$

$$x_{ij} = \sum_{k=1}^{s} q^k_{ij} \quad \forall i, j, k \quad (4)$$

$$0 \le q^k_{ij} \le R^k Y^k_{ij} \quad \forall i, j, k \quad (5)$$

$$Y^k_{ij} \ge Y^{k+1}_{ij} \quad \forall i, j, k \quad (6)$$

$$Y_{ij} \in \{0,1\}$$

$$x_{ij}, q_{ij} \ge 0 \text{ and integers}$$

The objective function (1) minimizes the total cost of transportation between all the supply and demand nodes. The first component represents variable cost of transportation and the second component takes into account the fixed costs for each segment incurred along given route. Variable Y^k_{ij} is defined as a binary variable and the rest of the variables q^k_{ij} and x_{ij} are restricted to integers. Constraints (2) and (3) are the usual supply and demand constraints. Constraint (4) disaggregates the quantity shipped between any source and destination pair (i, j) into smaller quantities for allocating to different segments. Constraint (5) ensures that the allocated quantities, represented by q^k_{ij}, always lie within the bounds prescribed for respective segments. To explain the importance of constraint (6), let us consider the following two cases.

Case I: When the fixed cost associated with each segment follows a monotonically increasing function i.e., $FC^1 < FC^2 < FC^3 \ldots < FC^s$.

Let us consider a hypothetical numerical example with the following values:

Table 1

Segment	Fixed Cost	LB - UB
1	100	0 - 100
2	100	101- 300
3	300	301 - 700
4	400	701 - 1300

From Table 1, $FC^1 = 100$; $FC^2 = 100$; $FC^3 = 300$; $FC^4 = 300$; $R^1 =100$; $R^2=200$; $R^3=400$; $R^4=600$

Let the quantity of shipment between any arc (i, j) be 800 i.e., $x_{ij} = 800$

Expanding constraints (4) and (5) result in the following equations:

$800 = q^1_{ij} + q^2_{ij} + q^3_{ij} + q^4_{ij}$

$0 < q^1_{ij} < 100Y^1_{ij}$ $\quad 0 < q^2_{ij} < 200Y^2_{ij}$ $\quad 0 < q^3_{ij} < 400Y^3_{ij}$ $\quad 0 < q^4_{ij} < 600Y^4_{ij}$

As the problem is a minimization problem, to satisfy the above constraints, the solver would assign the following values:

$q^1_{ij} = 100$; $q^2_{ij} = 200$; $q^3_{ij} = 400$; $q^4_{ij} = 100$; and $Y^1_{ij} = Y^2_{ij} = Y^3_{ij} = Y^4_{ij} =1$

This implies $Y^1_{ij} > Y^2_{ij} > Y^3_{ij} > Y^4_{ij}$ which further implies $Y^k_{ij} > Y^{k+1\,ij}$ for k = 1…….s

From the above explanation, it is clear that constraint (6) becomes redundant when the fixed costs for different segments follow a monotonically increasing function.

Case II: Fixed cost associated with each segment does not follow a monotonically increasing function

Let us take the same example with minor changes in the fixed costs.

$FC^1 = 100$; $FC^2 = 100$; $FC^3 = 50$; $FC^4 = 25$; $x_{ij} =800$; $R^1 =100$; $R^2=200$; $R^3=400$; $R^4=600$

As per constraints (4) and (5): $\quad 800 = q^1_{ij} + q^2_{ij} + q^3_{ij} + q^4_{ij}$

$0 < q^1_{ij} < 100Y^1_{ij}$ $\quad 0 < q^2_{ij} < 200Y^2_{ij}$ $\quad 0 < q^3_{ij} < 400Y^3_{ij}$ $\quad 0 < q^4_{ij} < 600Y^4_{ij}$

To take advantage of smaller fixed costs associated with segments 3 and 4, the solver would assign values as follows: $q^1_{ij} = 0$; $q^2_{ij} = 0$; $q^3_{ij} = 200$; $q^4_{ij} = 600$; and $Y^1_{ij} = 0$; $Y^2_{ij} = 0$; $Y^3_{ij} = Y^4_{ij} = 1$. It essentially implies that the solver completely ignores segments 1 and 2 and directly accounts for fixed costs pertaining to segments 3 and 4. Therefore, the presence of constraint (6) will overcome this problem by ensuring that fixed cost for segment *'k'* is accounted for only after accounting the fixed cost for its preceding segment *'k-1'*.

CHARACTERISTICS OF TEST DATASETS

To evaluate the efficacy of the proposed model, we generated random instances of SFCTP of varying sizes. The problem size is represented as $m \times n \times s$ where m and n represent the number of supply and demand nodes respectively and s represents the number of segments. The variable cost of transportation along every route is generated within the range [1, 10]. The fixed cost associated with first segment is generated in the range [50, 100] i.e., FC^1 lies in the range [50,100]. Assuming that the fixed costs follow a monotonically increasing function, for the rest of segments, FC^k is generated using the following expression.

$FC^k = FC^{k-1} \times MF$ where MF is a random number generated between 1 and 3 for k = 2….s (7)

The lower and upper bounds on quantities for different segments are computed using the following empirical formulae.

Lower bound of first segment i.e., $LB^1 = 0$ (8)

UB^1 = (minimum demand + maximum demand)/ (2* number of segments) (9)

For the rest of segments k = 2……s,

$LB^k = UB^{k-1} + 1$ (10)

$UB^k = UB^{k-1} \times MF$ where MF is a random number generated between 2 and 4 (11)

Choosing a proper value for upper bound on each segment is important to retain the stair cost structure of the objective function otherwise the problem will become simple FCTP. Take for example a two segment problem with demand in the range [25, 50]. Choosing UB^1 > maximum demand (50) will simply alter the structure of the problem to a FCTP with single fixed cost. Hence, using expressions (9) and (11), we ensure that majority of the UB^k values are deliberately chosen to be less than the maximum demand value. To validate the data generated, we cross-checked and found that the values generated by the formulae approximately match with that of the pricing structure of at least two logistics service providers. Other characteristics such as problem sizes, number of instances, ranges of supply and demand are tabulated in Table 2.

RESULTS

The models were solved using IBM CPLEX 12.5 optimizer by allowing them to run until the desired optimal criterion is attained or for 30 minutes, whichever is earlier on an Intel Core 2 Duo, 3 GHz processor with 2 GB of RAM. All times reported are in CPU seconds. The results are summarized in Table 3. By generating 5 instances of each problem size, a total of 90 problem instances have been generated. Barring three problem sizes i.e., 20 x 20 x 2, 20 x 20 x3, and 15 x 30 x 3, CPLEX was able to solve all the other problem instances to optimality within 30 minutes. In these three cases, the solver could not converge to optimality within the set computational time.

It can be observed that as the problem size increases (either number of demand nodes or number of segments), the computational time required to solve a given instance to optimality increases exponentially. This reinforces the fact that the problem is a NP-hard problem and very quickly becomes extremely difficult to solve beyond a particular problem size. Among all the different parameters, the number of segments appears to be the dominant factor that significantly affects the computational time. The reason being though the computational time increases with increase in the number of supply and demand nodes, it can be observed that a marginal increase in the number of segments i.e., from 2 to 3, drastically increases the computational time. This phenomenon can be attributed to the step structure of the objective function. CPLEX solver uses advanced branch-and-bound (B&B) and branch-and-cut (B&C) procedures to solve a given problem instance to optimality. Fathoming of nodes is an important process in B&B procedures that directly dictates the computational effectiveness. By increasing the number of steps of SFCTP, we believe, the B&B tree size also increases exponentially. This in turn makes the solver take more time to fathom nodes because of the availability of more choices where a superior solution could be found at each iteration.

Using the proposed MILP model, the highest problem size that can be solved to optimality within 30 minutes is 15 x 15 x 3. Beyond this problem size, CPLEX was not able to converge to optimality. For all problem sizes, the lower bounds generated using LP-relaxation consistently fell in the range of 60 – 80% away from the optimum. This inferior quality of LP bounds implies that the MILP model needs to be augmented with additional “valid inequalities” to strengthen the formulation.

CONCLUSIONS

Staircase cost structure is very common in the logistics industry. In many practical scenarios, staircase cost structures are simply replaced by quite crude linear approximations or one step fixed costs, due to the computational complexity associated with them. In this paper, we have proposed a MILP model for step-fixed charge transportation problem. After explaining the mathematical model in sufficient detail, we demonstrate its applicability on a small numerical example. Using extensive computational experiments, we found that the problem indeed is a “NP-super hard” problem with much “higher degree” of polynomial complexity. Using the proposed MILP model, we also learnt that 15 x 15 x 3 is the highest size of the problem instance that can be solved to optimality within 30 minutes of computational time. We believe the number of segments appears to be the dominant factor that significantly dictates the complexity of the problem and affects the computational time. Also, it is observed that the quality of lower bounds generated using LP relaxation is poor and the formulation needs to be strengthened by adding valid inequalities. This could be a good starting point for further research on this problem.

Table 2

Characteristics of test datasets

Problem size (m x n x s)	**Number of instances**	**Range of supply**	**Range of demand**
4 x 4 x 2	5	[25,50]	[25,50]
8 x 8 x 2	5		
10 x 10 x 2	5		
15 x 15 x 2	5		
20 x 20 x 2	5		
4 x 8 x 2	5	[50,100]	[25,50]
8 x 16 x 2	5		
10 x 20 x 2	5		
15 x 30 x 2	5		
4 x 4 x 3	5	[25,50]	[25,50]
8 x 8 x 3	5		
10 x 10 x 3	5		
15 x 15 x 3	5		
20 x 20 x 3	5		
4 x 8 x 3	5	[50,100]	[25,50]
8 x 16 x 3	5		
10 x 20 x 3	5		
15 x 30 x 3	5		

Table 3

Results obtained

Problem size (m x n x s)	Number of instances	Computational time (seconds)			Average % deviation of the lower bound
		Best case	Worst case	Average	
4 x 4 x 2	5	0.08	0.47	0.164	65.82
8 x 8 x 2	5	0.58	10.08	2.73	69.64
10 x 10 x 2	5	0.84	4.95	3.25	73.96
15 x 15 x 2	5	47.13	918.5	333.64	74.55
20 x 20 x 2	5	*	*	*	*
4 x 8 x 2	5	0.13	0.16	0.138	59.84
8 x 16 x 2	5	1.64	25.16	7.61	71.19
10 x 20 x 2	5	1.78	165.13	72.60	69.76
15 x 30 x 2	5	*	*	*	*
4 x 4 x 3	5	0.13	0.20	0.164	68.74
8 x 8 x 3	5	1.0	3.5	1.9	77.89
10 x 10 x 3	5	1.75	15.56	9.23	79.34
15 x 15 x 3	5	254.88	1768.36	1342.24	78.22
20 x 20 x 3	5	*	*	*	*
4 x 8 x 3	5	0.19	0.42	0.354	71.89
8 x 16 x 3	5	3.11	23.14	10.324	78.64
10 x 20 x 3	5	40.31	671.55	327.97	78.09
15 x 30 x 3	5	*	*	*	*

*- Non convergence because of exceeding stipulated time or memory exhaustion

Appendix A

Kowalski and Lev (2008) used the following numerical example to explain the working of their heuristics. We also consider the same example for demonstrating our MILP model.

Variable cost matrix

	D1 = 10	D2 = 30	D3 = 10
S1 = 15	1	3	1
S2 = 20	2	2	3
S3 = 15	2	1	2

Fixed cost matrix

10; 20	10;10	10;30
10;30	10;20	10;20
10;20	10;30	10;10

Bounds on each segment

Segment	**LB – UB**
1	0 – 5
2	*6 – M(=10000)*

Proposed MILP model:

Minimize

Z= 1x11+3x12+1x13+2x21+2x22+3x23+2x31+1x32+2x33+10Y111+20Y211+ 10Y112+10Y212+10Y113+30Y213+10Y121+30Y221+10Y122+20Y222+10Y123+ 20Y223+10Y131+20Y231+10Y132+30Y232+10Y133+10Y233

Subject to

Constraint set (2):

x11+x12+x13<=15

x21+x22+x23<=20

x31+x32+x33<=15

Constraint set (3):

x11 +x21+x31>=10

x12+x22+x32>=30

x13+x23+x33>=10

Constraint set (4):

x11-q111-q211=0

x12-q112-q212=0

x13-q113-q213=0

x21-q121-q221=0

x22-q122-q222=0

x23-q123-q223=0

x31-q131-q231=0

x32-q132-q232=0

x33-q133-q233=0

Constraint set (5):

q111-5Y111<=0

q112-5Y112<=0

q113-5Y113<=0

q121-5Y121<=0

q122-5Y122<=0

q123-5Y123<=0

q131-5Y131<=0

q132-5Y132<=0

q133-5Y133<=0

q211-9995Y211<=0

q212-9995Y212<=0

q213-9995Y213<=0

q221-9995Y221<=0

q222-9995Y222<=0

q223-9995Y223<=0

q231-9995Y231<=0

q232-9995Y232<=0

q233-9995Y233<=0

Constraint set (6):

Y111-Y211>=0

Y112-Y212>=0

Y113-Y213>=0

Y121-Y221>=0

Y122-Y222>=0

Y123-Y223>=0

Y131-Y231>=0

Y132-Y232>=0

Y133-Y233>=0

Binaries

Y111	Y211	Y112	Y212	Y113	Y213
Y121	Y221	Y122	Y222	Y123	Y223
Y131	Y231	Y132	Y232	Y133	Y233

Generals

x11 x12 x13

x21 x22 x23

x31 x32 x33

q111 q211 q112 q212 q113 q213

q121 q221 q122 q222 q123 q223

q131 q231 q132 q232 q133 q233

Solution provided by CPLEX solver is as follows:

x11=5 x12=5 x13=5 x22=20 x31=5

x32=5 x33=5 Y111=1 Y112=1 Y113=1

Y122=1 Y222=1 Y131=1 Y132=1 Y133=1

q111=5 q112=5 q113=5 q122=5

q222=15

q131=5 q132=5 q133=5

5	**5**	**5**
	20	
5	**5**	**5**

Objective function value Z=180

All other variables are 0.

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Abstract:

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Key Words/Phrases: Fixed charge; Transportation problem; Step function; MILP model; Staircase costs