Perfection of the Jury Rule by Rule-Reforming Voters

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Abstract

With no authority to change the constitution, a jury does the next best thing: it adopts the optimal rule given the constitution. At equilibrium, some jurors, called the rule reformers, vote independent of their information producing the second-best rule. The remaining jurors vote on the basis of their information enabling aggregation of the dispersed information. Arising from this asymmetric voting in a simultaneous jury game is an equivalence class of asymmetric strong Nash equilibria in pure strategies at which the information aggregation is at its best. Thus, the strategic act of rule reforming enables individual rationality to yield collective rationality. The coordination problem, as to which juror would play which role, can be solved by letting the jurors make a non-binding pre-play agreement specifying each juror's role; the agreement is self enforcing. The results hold for any voting rule, and any costs of erroneous conviction and acquittal.

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1. Introduction

The idea of a rule-reforming vote is the following. Suppose the constitution mandates that to convict a defendant in any criminal case, seven out of twelve jurors must vote guilty. In a specific case, however, the optimal rule is to convict by a unanimous vote. The jury can neither amend the constitution, nor defy it: the moment a simple majority of jurors votes to convict, the defendant is convicted. Lacking the freedom to adopt the optimal rule, the jury can do the second best thing: it can adopt the optimal rule given the constitution. An opening for such an action is provided by the jurors' freedom to vote as they please. Exercising the freedom, imagine five jurors vote innocent independent of their evidence, while the seven remaining jurors vote informatively, that is, on the basis of their information. Then the five jurors are said to be rule reformers: by voting innocent independent of the evidence, they have changed the rule; the defendant would be convicted only if the seven remaining jurors unanimously vote guilty. Indeed, the rule reformers have changed the rule for the better: unanimity among seven is closer to the optimal rule requiring unanimity among twelve than the mandated rule requiring a simple majority of twelve. More generally, rule reformers perfect the law: they acquit (resp. convict) when the probability of an erroneous conviction (resp. acquittal) under the mandated rule is excessive.

Rule reformers play one role: they change the rule, without changing the constitution, such that the amended rule is the optimal rule for the case under investigation. And given this optimal rule, the remaining jurors play one role: they vote informatively, enabling aggregation of the dispersed information whereby the jurors do better collectively than individually. The two roles are mutually exclusive.

In the literature, rule-reforming strategy is labeled uninformative strategy since a player voting independent of her signal conveys no information about it. Because uninformative voting occurs at optimal equilibria, the label creates an erroneous impression that an uninformative voter, by depriving the jury of her private information, enhances jury accuracy. The purpose of voting independent of the signal is not to conceal information but to change the rule. The main point is this: while each juror would contribute to jury accuracy by voting informatively, some of them would contribute more by optimizing the rule. Precisely for this reason, it is not optimal for all jurors to vote informatively unless the mandated rule is the optimal rule.

The theorems to prove are the following. First, in a simultaneous-voting jury game, when each juror plays her role, as a rule reformer or as an informative voter, then (a) each juror acts optimally: no juror (indeed no collection of jurors) would deviate to do something different, and (b) the jury would attain the true hypothesis with the highest probability, given the constitution. That is, the strategic act of rule-reforming (that is, voting independent of the evidence, and perhaps defying jury instruction) leads to an equilibrium at which individual rationality yields collective rationality; see Lemma 2. Stated more precisely, for a simultaneous ballot jury game, for any voting rule and for any costs of Type I and II errors, there exists a unique integer r* which defines an equivalence class of asymmetric pure-strategy strong Nash equilibria at which (a) r* jurors vote as rule reformers and the rest vote informatively, and (b) the probability that the jury ascertains the truth is maximized. Thus, each r*-equilibrium is efficient. See Theorem 1.

Second, the coordination problem, as to which jurors would play which role, can be solved by letting the jurors enter into a non-binding pre-play agreement specifying each juror's role -- the agreement would be self-enforcing so long as there are r* rule reformers: no coalition of jurors would find it optimal to deviate from the agreement; see Theorem 2.

Section 2 reviews the recent literature. Section 3, which examines the problem of an individual choosing between a pair of hypotheses, is a prelude to the main results of the jury game presented in Section 4. Section 4.1 presents the simultaneous jury game, and section 4.2 presents the sequential jury game. Conclusions follow.

2. Literature Review

The main point of the various jury theorems is that majority-rule voting aggregates decentralized information: if each juror has some distinct knowledge to correctly distinguish between a pair of hypotheses with probability $p > .5$, then a majority of the jury would do so with probability greater than p (Ladha, 1992, 1993). See Appendix A for a brief review of Condorcet's jury theorem and a jury theorem for dependent votes.

The jury theorems appear to assume that an individual's probability of being correct would be the same whether she decides the case alone or as a member of a group. Thus, learning through exchange of information among jurors is excluded. The exclusion implies that information aggregation is due solely to majority-rule voting. The assumption of no learning seems conservative in that if jurors did indeed learn from each other, they would only do better. Austen-Smith and Banks (1996) argue, however, that jurors may learn without communication, and such learning may destroy the information aggregation property of majority-rule voting.

Austen-Smith and Banks (1996) argue that each juror knows that her vote matters only if she is pivotal, that is, she creates or breaks a tie. So each juror pretends to be pivotal, and computes what others must observe to yield a tie. This "additional knowledge," the knowledge of the distribution of others' information compatible with a tie, is what each juror "learns" from the jury setting even though no juror communicates with another.

The learning has two damaging consequences. First, it may prompt a juror to change her solitary vote implying that the voting behavior assumed by the jury theorems, which ignores this additional knowledge, is not in equilibrium. Second, the additional knowledge, which is the same for each juror, may so dominate each individual's knowledge that each individual votes "ignoring" her private information. When a sufficiently large majority votes in this fashion, we have the undesirable equilibria at which no information aggregation occurs.

To establish the existence of an equilibrium at which information aggregation occurs, two precursors to this paper by Ladha, Miller and Oppenheimer (1994) and Ladha (1996) propose asymmetric pure strategy Nash equilibria at which the jury attains the truth with the highest probability under majority-rule voting. These papers complement Coughlan's (2000) work which assumes a different Constitution: first, there is a "straw poll" at which each juror's vote reveals her private information, and then there is the deciding vote at which the jurors vote unanimously in light of the Apooled@ information.

Another line of attack to establish information aggregation is by way of mixed-strategy equilibria. Myerson (1997) establishes a sequence of equilibria such that the probability that the group acts correctly approaches one as the size of the group approaches infinity. Feddersen and

Pesendorfer (1998), and Wit (1998) arrive at symmetric mixed-strategy equilibria at which majority-rule aggregates voter information, under the assumption that the agents with the same information vote the same way. Thus, they eliminate from consideration asymmetric equilibria on normative grounds. Yet, given that asymmetric equilibria are in pure strategies, strong Nash and Pareto superior to symmetric mixed-strategy equilibria (Ladha, 1996), the case of asymmetric equilibria appears compelling. A criticism of asymmetric equilibria has been that they Aleave the coordination problem unanswered (Wit, 1996, p. 375).

McLennan (1998) generalizes some of the above results by establishing that an optimal profile, in the sense of aggregating voter information, is Nash. In particular, whenever sincere voting (which may not be Nash) aggregates information, there would exist Nash equilibria that would do even better; a sincere vote is one which an individual would cast in solitude based only on her own information. McLennan, however, does not tell us whether optimal equilibria exist when sincere voting does not aggregate voter information. Sincere voting does not lead to information aggregation when each voter, voting sincerely, votes for hypothesis A (say) because each believes the prior probability that A is true or the cost of erroneously rejecting A is too high. As it turns out, for a sufficiently large jury, optimal equilibria with information aggregation do indeed exist at which some jurors vote as rule reformers and some informatively, even if each juror, acting in solitude, votes for A. Further, McLennan (1998) does not specify the optimal equilibria, hence, one cannot evaluate their plausibility.

A third line of attack to establish information aggregation is by way of conceiving the jury game as a sequential voting game. It is the line taken by Fey (1998), Dekel and Piccione (1997), and this paper. Dekel and Piccione (1997) show that the Nash equilibria of the simultaneous voting game, whether symmetric or asymmetric, are also Nash equilibria of the sequential voting game. Their main insight is that, each juror voting on the basis of being pivotal, computes what others must observe to yield a tie. From her private signal and the event that there is a tie, she obtains a total count of various signals which forms the basis of her vote. The knowledge of which predecessor voted for which hypothesis, which is the only information a juror learns from the sequential structure, does not change this count, and therefore the juror's vote, except when the outcome is already decided, in which case it does not matter how she votes. In particular, no information cascade can arise because the observed votes of the predecessors do not influence the successors.

Further, Dekel and Piccione (1997) show that the asymmetric equilibrium at which informative voters vote before the uninformative voters (rule reformers in the language of this paper) is not only Nash but also a sequential equilibrium of the sequential game. Unfortunately, it does not appear to be the natural way to play the game as discussed in the Introduction.

3. Voting in solitude: combining prior and costs of errors into a composite prior

An individual must distinguish between the null hypothesis A and the alternate hypothesis B, where each hypothesis could be a composite hypothesis. Her prior probability that the true hypothesis is A is ξ. She observes a signal s ε {α,β} pertaining to the true hypothesis as per the following distributions:

$$q_\alpha = P(s=\alpha|A)\ \varepsilon\ (.5,1], \text{ and } q_\beta = P(s=\beta|B)\ \varepsilon\ (.5,1].$$

Thus, A is more likely to transmit α than β, and B is more likely to transmit β than α.

After observing s, she selects an action from the set {a,b} to maximize her expected utility, where a (resp. b) is to accept hypothesis A (resp. B). Let the juror's utility from various action-hypothesis combinations be represented by

u_1 = u(a,A) = u(correctly accept the null hypothesis),

u_2 = u(b,B) = u(correctly accept the alternate hypothesis),

u_3 = u(a,B) = u(erroneously reject the alternate hypothesis; a Type II error), and

u_4 = u(b,A) = u(erroneously reject the null hypothesis; a Type I error).

It is assumed that $u_1 > u_4$ and $u_2 > u_3$. Also, without loss of generality, assume that the lowest possible utility is non-negative. The individual in question would choose a if and only if (denoted by $\Leftrightarrow$)

$$E[u(b,.|s] < E[u(a,.|s)$$

$$\Leftrightarrow u(b,A)P(A|s) + u(b,B)P(B|s) < u(a,A)P(A|s) + u(a,B)P(B|s)$$

$$\Leftrightarrow (u_2-u_3)\ P(B|s) < (u_1-u_4)\ P(A|s)$$

$$\Leftrightarrow P(B \mid s) < U\ P(A \mid s),$$

$$\text{where } U \equiv \frac{u_1 - u_4}{u_2 - u_3}. \qquad (1)$$

$\Leftrightarrow$ P(s | B) $(1-\xi)$ < U P(s | A) ξ (by Bayes' rule)

$$\Leftrightarrow \frac{1-\xi}{U\xi} < \frac{P(s \mid A)}{P(s \mid B)} \qquad (2).$$

Define $\pi = \dfrac{U\xi}{U\xi + (1-\xi)}$ so that $\dfrac{1-\pi}{\pi} = \dfrac{1-\xi}{U\xi}$ and (1) reduces to

$$\frac{1-\pi}{\pi} < \frac{P(s \mid A)}{P(s \mid B)} \qquad (3).$$

Equations (2) and (3) imply that the prior Pr (A) $= \xi$ and utilities of various errors (represented by U; equation 1) *can be* combined into a single variable π = prior Pr(A). In other words, (ξ, U) can be replaced by (π, U=1). The role of U is to rescale the prior probability ξ to yield π.

To see this, note that $\pi = \dfrac{U\xi}{U\xi + (1-\xi)} = \dfrac{\xi}{\xi + \frac{1-\xi}{U}}$. Evidently, $\pi = 0$ if $\xi = 0$, $\pi = 1$ if $\xi = 1$, and π is a strictly increasing in ξ because $\dfrac{d\pi}{d\xi} > 0$. Thus, U rescales ξ to yield π. It is also the case that $\pi = \xi$ if U = 1, $\pi > \xi$ if U > 1 and $\pi < \xi$ if U < 1, and $\dfrac{d\pi}{dU} > 0$. The derivatives $\dfrac{d\pi}{d\xi} > 0$ and $\dfrac{d\pi}{dU} > 0$ can be explained intuitively. A high π = prior Pr (A) makes a choice of A more likely

A high ξ = prior Pr (A) would make choice of A more likely just as a high U would; a high U means that the utility difference between voting a and b is more critical when the true state is A than when the true state is B. So when we combine ξ and U into π, it ought to be the case, as it indeed is, that $\dfrac{d\pi}{d\xi} > 0$ and $\dfrac{d\pi}{dU} > 0$.

We can state the above as a result:

Proposition 1. Suppose there are two hypotheses A and B. Let ξ be the prior probability of A and let $U \equiv \dfrac{u_1 - u_4}{u_2 - u_3} > 0$, where the utilities u_i's from various action-hypothesis combinations are as defined above. Define $\pi = \dfrac{U\xi}{U\xi + (1-\xi)}$. Then ($\xi$,U) can be replaced by ($\pi$, 1) such that A is chosen over B under (ξ,U) if and only if A is chosen over B under (π, 1).

To illustrate the simplification introduced by Proposition 1, let A be the hypothesis that a defendant is innocent, and B be the hypothesis that the defendant is guilty. Let $u_1 > u_2 > u_3 > u_4$, where u_1 = u(a,A) = u(acquitting the innocent), u_2 = u(b,B) = u(convicting the guilty), u_3 = u(a,B) = u(an erroneous acquittal; a Type II error), and u_4 = u(b,A) = u(an erroneous conviction; a Type I error). Proposition 1 states that we can set $u_1 = u_2 = 1$ (correct choices), and $u_3 = u_4 = 0$ (incorrect choices) provided that we replace ξ with $\pi = \dfrac{U\xi}{U\xi + (1-\xi)}$. Consequently, equation (2) in terms of (ξ, U) is replaced by equation (3) in terms of π. The constructed prior probability π serves dual purpose: it captures the effect of both ξ and U. Results in terms of π can be interpreted to allow for various combinations of ξ and U that yield π.

So to avoid carrying the term U, replace Π by π accompanied with $u_1 = u_2 = 1$ and $u_3 = u_4 = 0$ so that U = 1, and imagine π to be original prior probability that the true hypothesis is A. With U = 1, it follows from (1) that given s, the decision-maker would vote a if and only if P(B|s) < P(A|s). Thus, the advantage of using π, which embodies differences in various utilities, is that by choosing the more likely hypothesis with respect to the modified prior probability π, the decision maker would maximize her utility with respect to the original prior probability Π. Further, the restrictions to be placed on π could be interpreted as restriction on either Π, U or both.

From (3), we have

$$v(\alpha) = a \Leftrightarrow P(B|\alpha) < P(A|\alpha) \Leftrightarrow \frac{P(\alpha|B)}{P(\alpha|A)} < \frac{\pi}{1-\pi},$$

$$v(\beta) = b \Leftrightarrow P(A|\beta) < P(B|\beta) \Leftrightarrow \frac{\pi}{1-\pi} < \frac{P(\beta|B)}{P(\beta|A)}.$$

Definition. An action v is informative if the individual chooses "a" upon observing α, and b upon observing β.

A decision-maker would act informatively if and only if (iff) $P(A|\alpha) > P(B|\alpha)$ and $P(B|\beta) > P(A|\beta)$, that is, iff

$$\frac{P(s=\alpha\,|\,B)}{P(s=\alpha\,|\,A)} = \frac{1-q_\beta}{q_\alpha} < \frac{\pi}{1-\pi} < \frac{q_\beta}{1-q_\alpha} = \frac{P(s=\beta\,|\,B)}{P(s=\beta\,|\,A)}.$$

If $\pi = .5$, then (3) would hold for all $q_\alpha > .5$ and $q_\beta > .5$ leading to an informative action.

Definition. An action v is uninformative if it is independent of the signal.

A decision-maker would choose b uninformatively iff $P(B|\alpha) > P(A|\alpha)$, that is, iff $\pi/(1-\pi) \leq (1-q_\beta)/q_\alpha$, and would choose a uninformatively iff $P(A|\beta) \geq P(B|\beta)$, that is, iff $q_\beta/(1-q_\alpha) < \pi/(1-\pi)$.

Figure 1 shows the combined picture: if $\pi/(1-\pi)$ is $\leq (1-q_\beta)/q_\alpha$, choose b; is $\geq q_\beta/(1-q_\alpha)$,

choose b uninfo. | choose informatively | choose a uninfo.

$$\frac{1-q_\beta}{q_\alpha} \qquad\qquad \frac{q_\beta}{1-q_\alpha}$$

Figure 1

choose a; and lies in the middle, choose informatively.

Obviously, for a sufficiently high (resp. low) π, a decision-maker would choose a (resp. b) uninformatively, and be correct with probability π (resp. $1-\pi$). If she chose informatively, she would be correct with probability $p = q_\alpha\pi + q_\beta(1-\pi)$.

4. Simultaneous jury game

Imagine that the decision-maker of Section 3 belongs to a committee of n individuals, denoted by the set N = {1,...,n}. The committee must distinguish between the null hypothesis A and the alternate hypothesis B. All individuals have identical preferences and are as indicated in Section 3. Each i ε N has the same prior probability $\pi = P(A) < 1$ that the true hypothesis is A, where, as per Section 3, π also embodies various utility differences. Each i ε N observes a private signal s_i ε {α,β} pertaining to the true hypothesis as per the following distributions:

$$q_\alpha = P(s_i=\alpha|A)\ \varepsilon\ (.5,1], \text{ and } q_\beta = P(s_i=\beta|B)\ \varepsilon\ (.5,1],$$

such that the signals are conditionally independent.

The committee decides by the m-rule: adopt A if A gets m votes, adopt B if B gets n-m+1 votes. For a simple majority rule, $m = (n+1)/2$. Each individual votes to maximize her expected utility conditioned on whatever she knows at the time of voting. Clearly, each knows her own signal. Moreover, in a committee setting, each treats herself to be pivotal and knows the distribution of others' signals compatible with a tie. A voting strategy, v_i, for individual i maps his knowledge to {a,b}. A voting profile of the committee is a map v: $\{\alpha,\beta\}^n \rightarrow \{a,b\}$ defined by $v(s) = (v_1(s_1), ..., v_n(s_n))$.

<u>Definition</u>. A voting strategy is <u>informative</u> if the individual votes a (resp. b) upon observing private signal α (resp. β).

<u>Definition</u>. A voting strategy is <u>rule reforming</u> if the individual votes either a or b independent of her signal.[1]

Let r_a and r_b be the number of rule reformers who vote a and b, respectively. Let τ (for tie) be the event that a member is pivotal. Let $v_i(\tau,r_a,s_i)$ be i's vote when she observes s_i and is pivotal with r_a rule reformers voting a. Let $v_i(\tau,r_b,s_i)$ be i's vote with r_b rule reformers voting b. Define

[1]Given the utilities, an individual would act as a rule reformer only if it perfects the law.

$$G = \phi \frac{1 - q_\beta}{q_\alpha}; H = \phi \frac{q_\beta}{1 - q_\alpha}; \phi = \frac{(1 - q_\beta)^{m-1} q_\beta^{\,n-m}}{q_\alpha^{\,m-1} (1 - q_\alpha)^{n-m}};$$

$$d = \frac{q_\alpha}{1 - q_\beta}; e = \frac{1 - q_\alpha}{q_\beta}.$$

Together $q_\beta > .5$ and $q_\alpha > .5$ imply G > H, d > 1 and e < 1.[2] Hence,

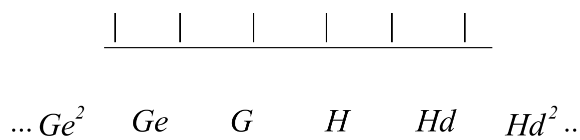

Figure 2

$G e^n < ... < G e^{r_b^*} < G e^{r_b^*-1} < ... < Ge < G < H < Hd < ... < H d^{r_a^*-1} < H d^{r_a^*} < ... < Hd^n$. See Figure 2.

If $r_a \geq m$ (resp. $r_b \geq n-m+1$), then a (resp. b) would be chosen independent of any evidence precluding any information aggregation. To ensure $r_a < m$, it is necessary that $Ge^{n-m} < \pi/(1-\pi)$, and to ensure $r_b < n-m+1$, it is necessary that $\pi/(1-\pi) < Hd^{m-1}$. The restriction on π is weaker than the usual requirement that in solitude each juror must vote informatively. Indeed, so long as m grows with n, the restriction on π will always be met for a sufficiently large jury.

There are three possibilities: π/(1-π) lies (a) between G and H, (b) at or to the right of H, and (c) at or to the left of G. To introduce Lemma 1, define a voting profile at r to be one at which there are r rule reformers and n-r informative voters.

Lemma 1. Suppose a committee of n members would adopt hypothesis A if and only if it gets m votes. Before voting, each member receives a private signal from {α,ß} such that $q_\alpha = P(\alpha|A) > .5$ and $q_\beta = P(\beta|B) > .5$. Each has prior probability π that the true hypothesis is A such that $Ge^{n-m} < \pi/(1-\pi) < Hd^{m-1}$. Then there exists a unique integer r^* such that the voting profile at r^* would

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[2] G has the following interpretation, G = P(B|A wins)/P(A|A wins). Likewise, for H.

constitute a pure-strategy Nash equilibrium. The integer r^* is defined as follows.

(1) If $G < \pi/(1-\pi) < H$, then $r^* = 0$ implying all jurors would vote informatively.

(2) If $H \le \pi/(1-\pi)$, then $r^* = r_a^*$ implying r_a^* rule reformers would vote a, where r_a^* is the smallest integer at which $Hd^{r_a^*-1} < \pi/(1-\pi) < Hd^{r_a^*}$.

(3) If $\pi/(1-\pi) \le G$, then $r^* = r_b^*$ implying r_b^* rule reformers would vote b, where r_b^* is the smallest integer at which $Ge^{r_b^*} < \pi/(1-\pi) < Ge^{r_b^*-1}$.

Proof. See Appendix B.

To see how the rule reformers change the rule, suppose $H \le \pi/(1-\pi)$ so that at equilibrium r_a^* rule reformers vote a. The effect is that to adopt hypothesis A, m-r_a^* of the remaining n-r_a^* jurors must vote a. Thus, the m-rule of the n-person jury is replaced by the (m-r_a^*)-rule of the (n-r_a^*)-person jury. As all (n-r_a^*) jurors vote informatively, it follows that the (m-r_a^*)-rule is the optimal rule of the (n-r_a^*)-person jury. Thus, the rule reformers optimize the constitutionally specified general rule before applying it to a specific case.

Lemma 2. For any given set of strategies of all others, a juror, under the conditions of Lemma 1, would arrive at the same decision whether she acts on the basis of being pivotal or to maximize the probability of committee success.

Proof. See Appendix B.

Lemma 2 states that an individually rational act, based on the assumption of being pivotal, is collectively rational. That is, incorporation of strategic considerations advances group interest.

Theorem 1. Let T(r) be the probability that a jury using an m-rule arrives at the correct decision when there are r rule reformers. Then at the r^* of Lemma 1, T(r) would attain its maximum.

Proof. See Appendix B.

Theorem 1 implies that at the pure-strategy Nash equilibrium at r^*, with m = (n+1)/2, the information aggregation is even better than that obtainable under the jury theorems. Further, Theorem 1 allows for public signals so long as the new common prior probability, updated in light of the public signal, is sufficiently moderate.

Theorem 2. The equilibrium corresponding to strategy profile r^* is a strong Nash equilibrium**.**
Proof. See Ladha (1998).

Recall, an equilibrium is strong Nash if no coalition, taking the actions of its complement as given, would together deviate. Strong Nash equilibria are both coalition-proof and Pareto optimal. An implication of Theorem 2 is that all members would coordinate on an equilibrium of Theorem 1 by making a non-binding pre-play agreement given a chance to communicate. The agreement would be both self-enforcing.

Example

Let n = 3, $\pi = .5$, $q_\alpha = .6$, and $q_\beta = .9$. Let m be the number of votes needed to adopt the null hypothesis A (the defendant is innocent). By (5), we have:

(1) For m = 1 (unanimity to convict): G = .84 and H = 11.39, so that $\pi/(1-\pi)$ ε (G,H), hence by Lemma 1, everyone would vote informatively. Thus, m = 1 is the optimal rule.

(2) For m = 2 (majority rule): G = .06, H = .84, Hd = 5.06, so that $\pi/(1-\pi)$ ε (H,Hd), hence by Lemma 1, one juror would vote a as a rule reformer, and two would vote informatively.

(3) For m = 3 (unanimity to acquit): G = .005, H = .06, Hd = .38 Hd^2 = 2.25, so that $\pi/(1-\pi)$ ε (Hd,Hd^2), hence by Lemma 1, two jurors would vote a as rule reformers, and one would vote informatively.

The first column of Table (*) lists all possible observed signals. The column "pooled sample," with everyone observing all signals, presents the outcomes that would be unanimously chosen. The numbers in parentheses are the number of votes for A and B, respectively. The next three columns, with each signal privately observed, specify outcomes under (1) m = 1, r_a = 0: unanimity to convict, it is the optimal rule given the parameters; (2) m = 2: majority rule with r_a = 1, that is, one juror (say, J1) voting a as a rule reformer; and (3) m = 3: unanimity to acquit with r_a = 2, that is, two jurors (say, J1 and J2) voting a as rule reformers. For comparison, the columns for m = 2 and m = 3 also specify after the slash the outcome if all voters were to vote informatively. The last column provides the probability of observing the sample in the first column if A was the true hypothesis.

Table (*)
Jury decisions under various rules

Private signal of J1,J2,J3 resp.	Pooled* Sample (#a,#b)	m=1, r_a=0 Optimal Rule (#a,#b) Unanimity to convict	m=2, r_a=1 Maj Rule (#a,#b) / outcome with all info	m=3, r_a=2 Unanimity to acquit (#a,#b) / outcome with all info	Pr(sample| A)
α, α, α	A (3,0)	A (3,0)	A (3,0)/A	A (3,0)/A	.216
α, α, β	A (3,0)	A (2,1)	A (2,1)/A	B (2,1)/B	.144
α, β, α	A (3,0)	A (2,1)	A (2,1)/A	A (3,0)/B	.144
β, α, α	A (3,0)	A (2,1)	A (3,0)/A	A (3,0)/B	.144
β, β, α	A (3,0)	A (1,2)	A (2,1)/B	A (3,0)/B	.096
β, α, β	A (3,0)	A (1,2)	A (2,1)/B	B (2,1)/B	.096
α, β, β	A (3,0)	A (1,2)	B (1,2)/B	B (2,1)/B	.096
β, β, β	B (0,3)	B (0,3)	B (1,2)/B	B (2,1)/B	.064

*The column refers to the case in which each juror observes all three signals.

Pooling, if feasible, is the best. The optimal rule, given that each juror's signal must remain private, is the second best though in this example, the optimal rule is as good as pooling: the outcome is the same under both schemes in all rows. Majority rule, with J1 acting as a rule reformer, is the third best: in the last but one row, the outcome differs from that of the pooled sample. Majority rule, with everyone voting informatively, column 4 after the slash (/), is obviously not as good: in the fifth through seventh rows, the outcomes differ from those of the pooled sample. Finally, unanimity to acquit, column 5, with J1 and J2 voting as rule reformers is worse than the majority rule. Unanimity to acquit with everyone voting informatively, column 5 after the slash, is the worst. Rule reforming obviously improves upon all voting informatively.

Clearly, majority rule over three jurors, with J1 voting as a rule reformer, is equivalent to an implicit rule that (i) excuses J1 from voting altogether, and (ii) imposes a super-majority rule on the remaining jurors: the outcome is B if and only if J2 and J3 vote b.

More generally, majority rule over n jurors, with k rule reformers, is equivalent to an implicit rule that (a) excuses k jurors from voting altogether, and (b) imposes a super-majority rule on the remaining n-k jurors: the outcome is B if and only if at least (n+1)/2 jurors vote b. When the number of rule reformers is at the optimum level k^* (it exists and corresponds to a minority of n voters by Theorem 1), information aggregation is at its (constrained) best as $(n-k^*)$ jurors vote informatively. Not surprisingly, the implicit rule, that imposes the super-majority rule on $n-k^*$ jurors as per (b), is the optimal rule for a jury of size $(n-k^*)$.

5. Conclusion

When the knowledge about the truth is imperfect and dispersed among people, and not amenable to direct pooling, it is necessary to adopt aggregation rules to select the better hypothesis.

The jury theorems specify conditions under which majority-rule voting aggregates decentralized information. But when majority rule is not the optimal rule, the voting behavior assumed by jury theorems is not Nash.

This paper offers an equivalence class of asymmetric Nash equilibria in pure strategies at which the probability of attaining the truth under the mandated rule of voting is maximized. The equivalence class is characterized by a unique integer r^* which represents the number of voters acting as rule reformers. The rule reformers lead the remaining voters, who vote informatively, to the optimal rule subject to the mandated rule.

Although the number of rule reformers is uniquely determined by the model parameters, the coordination problem as to who will play which role needed to be solved. It is solved by letting the jurors arrive at a non-binding pre-play agreement that is self enforcing. Once the rule is reformed to the optimal rule, the remaining voters vote informatively maximizing the probability of attaining the true hypothesis.

Appendix A

Condorcet's Jury Theorem

If each of the n members of a committee votes correctly with probability $p > .5$, and if the votes are statistically independent, then a committee majority would be correct with probability $P_n > p$. Further, P_n will monotonically approach 1 as n approaches infinity.

The condition $p > .5$ requires each member to have some knowledge of the truth.[3] Independence requires members' knowledge to be non-overlapping. The theorem states that majority rule voting aggregates member information, and that the larger the committee, the better.

Certain shortcomings of CJT are obvious. First, the source of uncertainty is unspecified. Second. the assumption of independence precludes common information, and committee deliberation. The following theorem for dependent votes partially addresses both shortcomings by (a) framing the jury choice between a pair of hypotheses; thus, uncertainty arises from the sample (Ladha, 1996), and (b) replacing independence by conditional independence.[4] For j = A,B, let

$\pi = P(A)$ be the prior probability that hypothesis A is true,

$q_j = P(j|j)$ = P(each member votes correctly given j),

$P_{n,j}$ = P(A majority of n members votes j|j).

Let the votes be independent conditional on the true hypothesis. Then, p = P(a member is correct)

= P(select A|A) P(A) + P(select B|B) P(B)

$= q_A\pi + q_B(1-\pi)$.

And P_n = P(a majority is correct) $= P_{n,A}\,\pi + P_{n,B}\,(1-\pi)$.

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[3]The condition that members have the same $p > .5$ is not necessary, the average probability $\bar{p} > .5(1+1/n)$ would suffice (Boland, 1989).

[4]For additional results with voter dependency, see Shapley and Grofman (1984), and Ladha (1992, 1993).

Note that unconditionally the votes are correlated: P(two members vote correctly) = P(both select A|A) P(A) + P(both select B|B) P(B) = $q_A^2 \pi + q_B^2 (1-\pi) \neq (q_A \pi + q_B (1-\pi))^2$, unless $\pi = 0$ or 1, or $q_A = q_B$.

Jury theorem for dependent votes

Suppose conditional on the true hypothesis, each member votes correctly with probability greater than .5, that is, both q_0 and $q_1 > .5$, and the votes are independent. Then a majority of the committee will be correct with probability $P_n > p$. Further, P_n monotonically increases in n.

Proof. To prove that $P_n > p$, and that P_n monotonically increases in n, it is sufficient to prove that $P_{n,j} > q_j$ and that $P_{n,j}$ monotonically increases in n for j = A,B. The result for each j follows directly from the CJT.

The theorem states that even if the members' knowledge is overlapping, a committee would do better than any subcommittee of it, and the larger the committee the better.[5] Thus, voter dependency does not preclude information aggregation by majority rule.

[5]For a version of the theorem in terms of probabilities of Type I and II errors, $1-q_0$ and $1-q_1$, respectively, see Ladha (1996).

Appendix B

Proof of Lemma 1.

$v_i(\tau,r_a,s_i)$ = a if and only if $E[u(b,.|\tau,r_a,s_i] < E[u(a,.|\tau,r_a,s_i)$. By imitating the steps leading up to inequalities (2), we have at r_a,

$$v_i(\tau,r_a,\alpha)=a \text{ iff } P(A|.)>P(B|.) \text{ iff } \frac{P(\tau,r_a,\alpha|B)}{P(\tau,r_a,\alpha|A)}<\frac{\pi}{1-\pi},$$

$$v_i(\tau,r_a,\beta)=b \text{ iff } P(B|.)>P(A|.) \text{ iff } \frac{\pi}{1-\pi}<\frac{P(\tau,r_a,\beta|B)}{P(\tau,r_a,\beta|A)}.$$

When there is a tie, with r_a rule reformers voting a and the rest voting informatively, voter i has the "additional information" that $m-1-r_a$ members have observed α, and n-m members have observed β. Therefore, at r_a

$$\frac{P(\tau,r_a,\alpha|B)}{P(\tau,r_a,\alpha|A)}=\frac{P\big((m-1-r_a)\alpha,(n-m)\beta,s_i=\alpha|B\big)}{P\big((m-1-r_a)\alpha,(n-m)\beta,s_i=\alpha|A\big)}\equiv G\,d^{r_a},$$

where

$$\frac{P(\tau,r_a,\beta|B)}{P(\tau,r_a,\beta|A)}=\frac{P\big((m-1-r_a)\alpha,(n-m)\beta,s_i=\beta|B\big)}{P\big((m-1-r_a)\alpha,(n-m)\beta,s_i=\beta|A\big)}\equiv H\,d^{r_a},$$

As Figure B-1 shows, (B-1), (B-2) and (B-3) imply that at r_a, if $\pi/(1-\pi)$

(a) is at or to the left of Gd^{r_a}, then vote b independent of the signal;[6]

$$G=\phi\frac{1-q_\beta}{q_\alpha};\ H=\phi\frac{q_\beta}{1-q_\alpha};\ \phi=\frac{(1-q_\beta)^{m-1}q_\beta^{\ n-m}}{q_\alpha^{\ m-1}(1-q_\alpha)^{n-m}};\ and\ d=\frac{q_\alpha}{1-q_\beta}>1.$$

(b) is at or to the right of Hd^{r_a}, then vote a independent of the signal, there are too few rule reformers voting a;

(c) lies in the middle, then vote informatively, the number of rule reformers is just right.

[6] Too many rule reformers are voting a, and if it indeed happens then the best is to cancel an a vote by voting b independent of one's private signal.

vote b as a rule reformer | vote informatively | vote a as a rule reformer

Gd^{r_a} $\qquad\qquad$ Hd^{r_a}

Figure B - 1

Similarly, we have at r_b,

$$v_i(\tau, r_b, \alpha) = a \text{ iff } P(A|.) > P(B|.) \text{ iff } G\,e^{r_b} < \frac{\pi}{1-\pi},$$

$$v_i(\tau, r_b, \beta) = b \text{ iff } P(B|.) > P(A|.) \text{ iff } \frac{\pi}{1-\pi} < H\,e^{r_b},$$

where $e = (1-q_\alpha)/q_\beta < 1$. As Figure B-2 shows, (B-4) implies that at r_b, if $\pi/(1-\pi)$

(a) is at or to the left of Ge^{r_b}, then vote b independent of the signal, there are too few rule reformers voting b;

(b) is at or to the right of He^{r_b}, then vote a independent of the signal;[7]

vote b as a rule reformer | vote informatively | vote a as a rule reformer

Ge^{r_b} $\qquad\qquad$ He^{r_b}

Figure B - 2

(c) lies in the middle, then vote informatively, the number of rule reformers is just right.

Both Figure B-1 reduces to Figure B-3 at $r_a = 0$, and Figure B-2 reduces to Figure B-3 at $r_b = 0$.

[7]Too many rule reformers are voting b, and if it indeed happens then the best is to cancel a "b" vote by voting a independent of one's private signal.

vote b as a rule reformer | vote informatively | vote a as a rule reformer

G *H*

Figure B - 3

At $r_a = r_b = 0$, that is with all others voting informatively, there are exactly three possible cases to consider: (a) π/(1-π) is between G and H; (b) π/(1-π) is at or to the right of H; and (c) π/(1-π) is at or to the left of G; see Figure 4.

Case (a). If π/(1-π) is between G and H, then juror i would vote informatively yielding

| | | | | |

Ge^2 *Ge* *G* *H* *Hd* Hd^2

Figure B - 4

the equilibrium at which everyone votes informatively. The m-rule is the optimal rule.

Case (b). If π/(1-π) is at or to the right of H, then a juror, contemplating how to vote, would vote a independent of her signal leading to $r_a = 1$. At $r_a = 1$, that is with one juror voting a independent of her signal, if π/(1-π) exceeds Hd, Figure B-4, then once again a juror, contemplating how to vote, would vote a independent of her signal leading to $r_a = 2$. And so on till either A obtains m votes and wins, or π/(1-π) is less than Hd^r for some r.

Let r_a^* be the smallest integer at which $Hd^{r_a^*-1} < \pi/(1-\pi) < Hd^{r_a^*}$. Because $Gd^{r_a^*} < Hd^{r_a^*-1}$, it follows that $Gd^{r_a^*} < \pi/(1-\pi) < Hd^{r_a^*}$ implying that at r_a^*, any juror, contemplating how to vote, would vote informatively. Thus, at equilibrium there will be exactly r_a^* jurors voting a independent of their signals. Clearly, r_a^* is unique.

Case (c). If π/(1-π) is at or to the left of G, then a juror, contemplating how to vote, would vote b independent of her signal leading to $r_b = 1$. At $r_b = 1$, that is with one juror voting b independent of her signal, if π/(1-π) is less than Ge, Figure B-4, then once again a juror, contemplating how to vote, would vote b independent of her signal leading to $r_b = 2$. And so on till either B obtains m votes and wins, or π/(1-π) is less than Ge^r for some r. Let r_b^* be the smallest integer at which $Ge^{r_b^*} < \pi/(1-\pi) < Ge^{r_b^*-1}$. Because $Ge^{r_b^*-1} < He^{r_b^*}$, it follows that $Ge^{r_b^*} < \pi/(1-\pi) < He^{r_b^*}$ implying that at r_b^*, any juror, contemplating how to vote, would vote informatively. Thus, at equilibrium there will be exactly r_b^* juror voting b independent of their signals. Clearly, r_b^* is unique.

Proof of Lemma 2.

Without loss of generality, let G < π/(1-π) so that rule reformers, if any, vote a. Let $T(r_a)$ be the probability that the jury selects the true hypothesis when r_a rule reformers vote a. Then

$T(r_a)$ = [P(# of a votes=m-1|A) + P(# of a votes ≠ m-1|A)] P(A)

+ [P(# of b votes=n-m|B) + P(# of b votes ≠ n-m|B)] P(A).

Clearly, a juror, say i, would change $T(r_a)$ only when she is pivotal; when she is not pivotal she does not affect the outcome and hence has no influence on $T(r_a)$. When she is pivotal she knows the number of α's and β's that the jury collectively possesses. On the basis of this knowledge she opts for the more likely hypothesis. It is precisely this act of choosing the more likely hypothesis that increases $T(r_a)$. Hence, voting based on the assumption of pivotality maximizes the probability that the jury makes the correct choice.

Proof of Theorem 1.

As in the proof of Lemma 2, let G < π/(1-π) so that rule reformers, if any, vote a. Let $r = r_a$ in the remainder of this proof. Consider juror i who votes informatively at r and is contemplating whether to vote a as a rule reformer. She would do so if it increases the probability of jury accuracy, that is, if T(r+1) > T(r). By Lemma 2, T(r+1) > T(r) if, holding constant the strategies of all other voters, juror i finds that P(A| τ ,r,β) > P(B| τ ,r,β), that is, if by (B-1) and (B-3), π/(1-π) > H d^r.

Clearly, d^r strictly increases in r because d > 1. Let a be a real, possibly negative, number at which π/(1-π) = H d^a. Then, T(r+1) > T(r) if and only if,

$$\frac{\pi}{1-\pi} = H\, d^{a} > H\, d^{r} \ \_\ a > r \ \_\ r^{*} > r,$$

where $r^* \geq a$ is the smallest non-negative integer, and r ε [0,m-1] is an integer; the symbol __ stands for "if and only if." It follows from (B-5) that T(r) strictly increases with r in $[0,r^*]$, attains its <u>unique</u> maximum at $r = r^*$, and strictly decreases with r in $[r^*,m-1]$. Moreover, $T(r^*) \geq T(m-1)$, and if it can be shown that $T(m-1) > T(r \geq m) = \pi$, then $T(r^*)$ would be the maximum over all r in [0,n]. Now $T(m) < T(m-1)$ iff $\pi/(1-\pi) < Hd^{m-1}$ which is true by the restriction imposed on π. In particular, from (B-5)

$T(r^*) > T(r^*+1)$ for any r^* ε [0,m-1], and

$T(r^*) > T(r^*-1)$ for any r^* ε [1,m-1].

The first (second) inequality states that holding constant the strategies of all others at r^*, if a juror voting informatively (rule reformingly) unilaterally switches her strategy, it would make her worse off by reducing the probability that a majority is correct. Together, the two inequalities imply that the strategy profile at r^* constitutes a Nash equilibrium.

By the strict monotonicity of T(r), with r ε [0,m-1], it follows that no r in [0,m-1], $r \neq r^*$, could correspond to a Nash equilibrium as there will exist a juror with an incentive to switch her strategy at such an r. This completes the proof.

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