Non-linearities in Emerging Financial Markets: Evidence from India

IIM Kozhikode Society & Management Review
2(2) 165–175

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Abstract

Efficiency and predictability of financial markets are inherently linked to the statistical properties of market indicators. While many papers have researched non-linearities in developed financial markets, this article examines chaotic dynamics in daily data taken from four financial markets in India, an emerging economy. The financial markets considered are the stock market, the foreign exchange market, the money market and the bond market. We employ four tests for detecting non-linearities, such as the BDS test on raw data, the BDS test on pre-whitened data, Correlation Dimension test, and Brock's Residual test. We find that the market indicators are not characterized by white noise or GARCH processes. Our results do not provide evidence for chaos but indicate the presence of other non-linear deterministic processes. These findings have important implications for investments in these markets.

Keywords

Chaos, non-linear dynamics, BDS test, correlation dimension, financial markets

Introduction

The bedrock of numerous studies in modern investment analysis is the efficient market hypothesis which emphasizes the non-predictability of financial markets. This has led to extensive empirical research into the time-series properties of financial markets employing mostly linear and non-linear probabilistic models. Recently, evidence of a phenomenon called deterministic non-linearity (one example of which is chaotic behaviour) in financial markets has raised the possibility of short term predictability of prices and returns of financial products (see Copeland Abhyankar, & Wong, 1995; McKenzie, 2001; Scheinkman & LeBaron, 1989). While non-linearity—stochastic or deterministic—is fairly well researched in developed economies, the evidence from emerging economies is scarce. Unlike developed markets, emerging markets may not adhere to the assumptions of standard theoretical models and hence require separate investigation (Bekaert & Harvey, 2002). Caballe, Jarque, and Michetti (2006) assert that emerging economies, with intermediate levels of financial development, are more likely to exhibit chaotic dynamics than economies with very developed or undeveloped financially markets. It is in this context that this paper investigates the types of non-linearity present in emerging economy financial markets with data from India.

While time-series studies of emerging financial markets have frequently employed stochastic models such as Auto Regressive Integrated Moving Average (ARIMA) and Generalized Auto Regressive Conditional Heteroscedasticity (GARCH), the possibility of chaotic dynamics remains scarcely researched. The presence of chaotic dynamics in financial market prices has interesting implications. Chaotic behaviour can potentially explain fluctuations in the price series that otherwise appear to be random processes (Trippi, 1995). The defining characteristic of chaotic processes is sensitive dependence on the initial conditions which means that two points in a system that have very similar initial conditions may exhibit substantially different trajectories. If a financial market exhibits such a complex non-linear pattern, then notwithstanding its apparent randomness, investors would be interested in exploiting such patterns. Such a characterization of the markets would also be useful for regulators in formulating

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their policies and for researchers in improving their understanding of how financial markets actually behave. Thus, in this article we are primarily concerned with this aspect of the empirical literature on non-linearity, such as the detection of chaotic dynamics.

The search for chaos in financial markets has been mostly restricted to stock markets and that too in developed economies. However given the very different institutional features of financial markets in emerging economies, it is important to separately explore the possibilities of such markets exhibiting chaotic behaviour. According to Caballe *et al.* (2006), emerging economies are typically characterized by credit constraints that engender the possibility of chaotic dynamics in financial markets. Moreover, financial markets in emerging economies are relatively less mature or deep as compared to those in developed countries, and the implications of complex non-linear behaviour could be significant for traders, institutional investors as well as policy makers in such countries. Therefore, given the recent increase in interest in emerging financial markets, it is interesting to investigate whether such markets are characterized by non-linearity of the chaotic type. Deeper understanding of the prices of financial instruments in emerging markets is also important for pricing of derivatives based on these instruments (Opong, Mulholland, Fox, & Farahmand, 1999). This is especially important given that the derivatives markets in emerging economies are relatively new and still evolving.

In this backdrop, this article examines financial markets of India, an emerging economy, and attempts to detect non-linearity of the chaotic type by employing four standard tests from the chaos literature. These are the BDS (Brock, Dechert & Scheinkman, 1996) test on raw data, the BDS test on pre-whitened data, Correlation Dimension test and Brock's residual test. We utilize daily price data from stock, foreign exchange (forex), money and bond markets over a fairly long time horizon. While this is one of the very few studies on chaos for an emerging economy, it is to our knowledge the first study that analyzes several financial markets of the same country employing a battery of tests for chaos.

The results from our empirical analysis are summarized as follows. The data from Indian financial markets are characterized neither by white noise nor by GARCH processes. Further we do not find evidence of chaos but our findings indicate the possibility of deterministic non-linear behaviour. This implies the rejection of efficient market hypothesis and indicates scope for hedging and gains through speculation. Also given the non-chaotic deterministic nature of most of these markets it should be possible to forecast into the long term with minimal loss of predictability, provided the appropriate model can be developed. However our focus is not on identifying the best model to explain the behaviour of the markets. Rather we restrict our analysis to testing for non-linear processes in general and chaotic processes in particular. The rest of the paper is organized as follows. The second section in the article discusses the extant evidence in the empirical literature. The third section outlines the tests used in the paper for detection of chaos, followed by the fourth section that introduces the data used. The fifth section presents and discusses the empirical results. The last and sixth section concludes the article or study.

Existing Evidence

The study by Scheinkman and LeBaron (1989) was one of the first attempts at applying the tools of non-linear dynamics to stock market returns. The authors found some evidence for chaos in stock return data from US markets. Since then, empirical tests on the existence of deterministic chaos in economic series have proliferated (see Sayers [1991] for a survey) and there has been a surfeit of studies of chaos in stock markets in various countries. Willey (1992) examined daily price data from the S&P Composite Index and NASDAQ100 Index but failed to detect any deterministic chaos. Pandey, Kohers, and Kohers, (1998) did not find low-dimensional deterministic chaos in the major US and European stock markets. Copeland *et al.* (1995) provided evidence for non-linearity in the FTSE-100 Index and employed GARCH models to explain some of the non-linearity. McKenzie (2001) provided evidence of non-chaotic non-linearity in the stock markets of 10 developed countries viz. Australia, Canada, France, Germany, Hong Kong, Japan, Singapore, Switzerland, UK and US. Serletis and Shintani (2003) failed to detect chaotic dynamics in US stock markets. More recently, Chappell and Panagiotidis (2005) and Mishra, Sehgal, and Bhanumurthy (2011) did not find evidence for chaotic behaviour in stock indices of Greece and India respectively.

Compared to the number of studies of chaos in stock markets, research for chaos in the other financial markets is scarce. Hsieh (1989) and Kugler and Lenz (1993) were one of the initial studies of non-linearity in exchange rates, and Diebold and Nason (1990) carried out non-parametric estimations of non-linear models of exchange rates. A study on interest rate and foreign exchange market was conducted by Wagner and Mahajan (1999) where interest rates for 11 countries (viz. Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, U.K. and U.S.) and foreign exchange rates for 10 currencies for January 1974 to November 1991 were studied. Using the BDS statistic and a correlation dimension analysis, the paper concluded that although forward premium is statistically significant and not random, the markets have become more complex and consequently are less predictable using forecasting tools. More recently, Scarlat, Stan and Cristescu (2007) detected deterministic chaos in the exchange rate of Romania.

While there are a number of studies of chaos in developed financial markets, this article is the first to study stock, forex, money and bond markets of an emerging economy, India. The focus of our study is to understand the inherent order of the Indian financial markets by trying to detect non-linearity in the form of deterministic chaos.

Empirical Strategy and Data

Our empirical strategy consists of a number of tests to detect chaotic behaviour in time series data of different financial markets. Hsieh (1991) and Trippi (1995) provide comprehensive expositions of the tests for detecting chaos in financial time series. In this paper we take the following approach: First, we employ the BDS test to ascertain if the raw data is generated by a random process. The null hypothesis of this test is that the data are random, i.e., independently and identically distributed. In case the null of randomness gets rejected, the series may be either linear stochastic, non-linear stochastic or non-linear deterministic. Next, we filter the raw data using the appropriate Auto Regressive (AR)-GARCH model and run a BDS test on the standardized residuals.[1] If the null hypothesis is not rejected using the residuals data, it suggests that the series may be following the fitted AR-GARCH process. Rejection on the other hand indicates that the series follows a non-linear deterministic process which may be chaotic. In the next stage of the analysis, we investigate the raw data series using the Correlation Dimension test. Here one estimates a number known as 'correlation dimension', which roughly measures the space filled by a string of data (Hsieh, 1991) and corresponds to the dimensionality of the space occupied by it. If the correlation dimension plot has a slope of less than 1 and plateaus at some point then it indicates the presence of low dimensional chaos in the series. In the last stage of the analysis we filter the raw data again using appropriate AR-GARCH model and use the filtered series to re-estimate the correlation dimension plot. This is known as Brock's residual test. If this plot coincides with the correlation dimension plot using raw data, then it is further evidence of chaos. Appendix 1 provides the technical details of each of these tests.

We now describe the data employed in this study. The details of the series studied and sources of data are provided in Table 1. The starting point of each series was chosen by going back as far as data was available from publicly available official sources and the data end on the last trading day of the financial year 2012–13. We studied

Table 1. Description of Data

Market	Stock Market (National Stock Exchange)	Forex Market	Money Market	Bond Market
Indicator	NIFTY (S&P CNX Nifty Index of 50 stocks)	U.S. Dollar – Indian Rupee Exchange Rate	Inter-bank overnight call/ notice money rate (weighted average)	Portfolio YTM from the NSE-Government Securities Index
Period	3 January 1994 to 28 March 2013	25 August1998 to 28 March 28, 2006	1April 2005 to 31 March 2013	1 January 1997 to 28 March 2013
Frequency	Daily	Daily	Daily	Daily
Mean	0.015	–0.003	6.209	9.087
Std. Dev.	0.706	0.170	2.755	2.118
Skewness	–0.121	0.040	6.261	0.494
Kurtosis	9.411	9.753	103.957	2.210
Jarque–Bera test (*p*-value)	8181.758 (0.000)	6719.771 (0.000)	1128931.984 (0.000)	291.531 (0.000)
Number of observations	4,772	3,537	2,618	4,370
Source	National Stock Exchange	Reserve Bank of India	Reserve Bank of India	National Stock Exchange

data from the two main Indian stock markets, viz. the National Stock Exchange (NSE) and the Bombay Stock Exchange (BSE). For NSE, we considered the main NIFTY (S&P CNX NSE Fifty) index and took its daily logarithmic return figures, for the period 3 January 1994 to 28 March 2013. For BSE, we considered its main index SENSEX (BSE Sensitive Index) and took its daily logarithmic return figures, for the period 1 January 1990 to 28 March 2013. The results for the BSE SENSEX were similar to the case of NIFTY and hence are not reported to save space. 2 For the forex market, we used the daily U.S. dollar–rupee exchange rate (in logarithmic returns form) for the period 25 August 1998 to 28 March 2013. For the money market, we used the weighted average of daily inter-bank overnight call/ notice money rate for the period 1 April 2005 to 31 March 2013. For the bond market, we used the portfolio Yield-To-Maturity from the NSE-Government Securities Index for the period 1 January 1997 to 28 March 2013. Summary statistics of each series are presented in Table 1. In all cases, Kurtosis is very high and the Jarque-Bera tests clearly reject the null of normality. These statistics point towards the possibility of non-linear dynamics in the data.

Results and Discussion

The results of our analysis are now presented in the order of the financial markets considered. We begin with the stock market.

Stock Market

The results from the BDS test on the NIFTY returns are provided in Table 2. The null hypothesis (that is, original series comes from a random data generating process) gets rejected for all measured dimensions, at 1 per cent level of significance. This suggests that the NIFTY return series is non-white (that is, not independently and identically distributed). The results from the BDS test on AR-GARCH filtered residuals of NIFTY returns are also provided in Table 2. The null hypothesis (i.e., that the residual series is generated by a random data generating process) gets rejected for all dimension values measured. Since the AR-GARCH residual series is non-white, AR-GARCH does not appear to be an adequate model that can explain the NIFTY returns. This indicates the possibility of non-linear deterministic processes including chaos, for which the subsequent tests are necessary.

Results from the Correlation Dimension test on NIFTY returns are presented in Figure 1(a). The graph shows the values of correlation dimension for increasing embedding dimensions. It can be noted that the correlation dimension value does not plateau for increasing embedding dimensions as would happen for a series that is chaotic. This suggests that the NIFTY return series is not chaotic. Furthermore the slope of the graph is much less that 45 degrees and the correlation dimension stays well below the embedding dimension even as the embedding dimension rises (correlation dimension equals 8.79, 9.66, 10.70 for embedding dimensions 10, 11, 12, respectively).

Table 2. *Z*-statistics Based on BDS Test for Stock, Forex, Money and Bond Markets

Embedding Dimension	NIFTY		Exchange Rate		Call Money Rate		Bond YTM	
	Raw	Filtered	Raw	Filtered	Raw	Filtered	Raw	Filtered
2	16.64	16.25	27.44	27.49	134.42	10.68	225.96	28.35
3	20.41	20.49	35.14	35.24	143.00	15.46	244.21	32.20
4	23.71	23.89	40.97	41.05	154.06	18.10	266.29	35.15
5	26.37	26.66	46.71	46.77	169.92	20.20	297.52	38.20
6	29.22	29.60	53.19	53.24	191.33	22.45	339.76	41.54
7	32.29	32.73	60.81	60.86	219.37	24.38	395.35	45.43
8	35.38	35.87	69.91	69.95	255.40	26.94	467.56	50.23
9	38.78	39.34	80.76	80.86	301.52	29.63	560.83	56.16
10	42.51	43.13	94.07	94.20	360.45	32.65	681.11	63.52
11	46.78	47.46	110.36	110.53	435.59	36.43	836.29	72.08
12	51.65	52.44	130.67	130.87	531.45	41.18	1036.85	82.28

Notes: *Z*-statistics have been presented after dividing the BDS test statistics with the standard errors. All the values are statistically significant at 1% significance level.

This suggests that the NIFTY return series is not stochastic and may be deterministic. Thus, our results point towards what could be a non-chaotic deterministic process in the stock market data. To conduct Brock's residual test, an AR-GARCH model was fitted into the NIFTY return data to obtain the respective standardized residuals. Results from the Correlation Dimension test of the residuals are plotted in Figure 1(b). That the series is not chaotic can

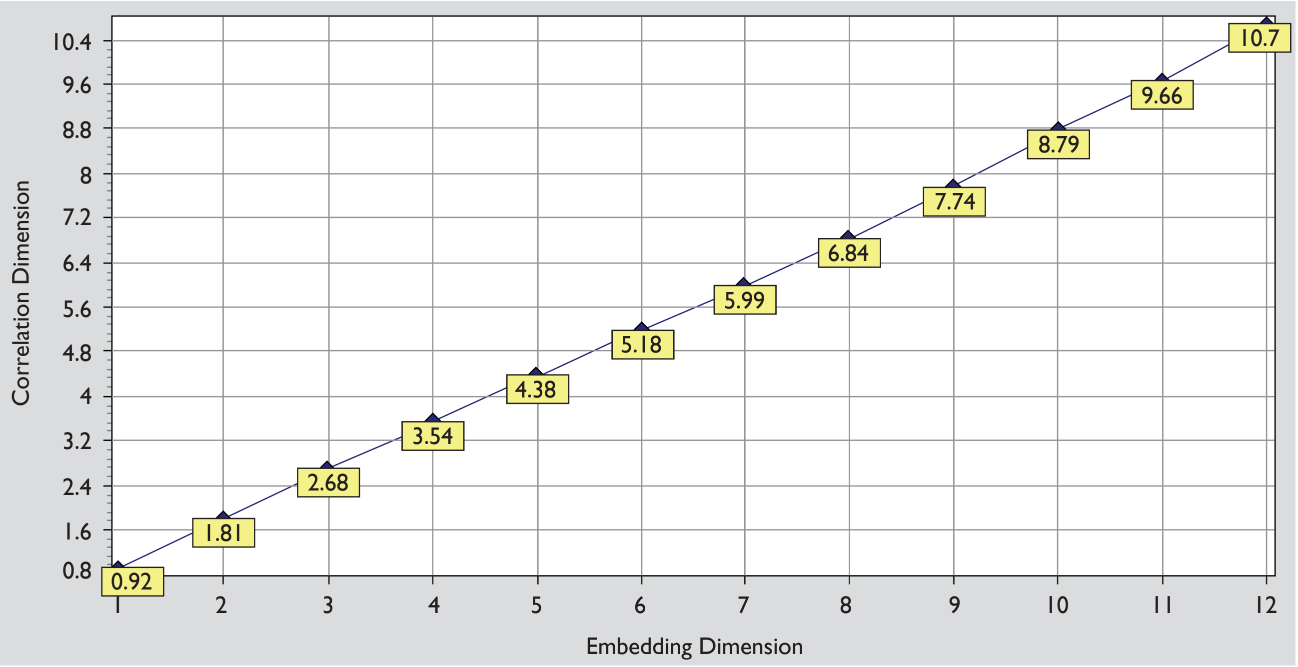

Figure 1(a). Plot of Correlation Dimension for Increasing Embedding Dimensions for NIFTY

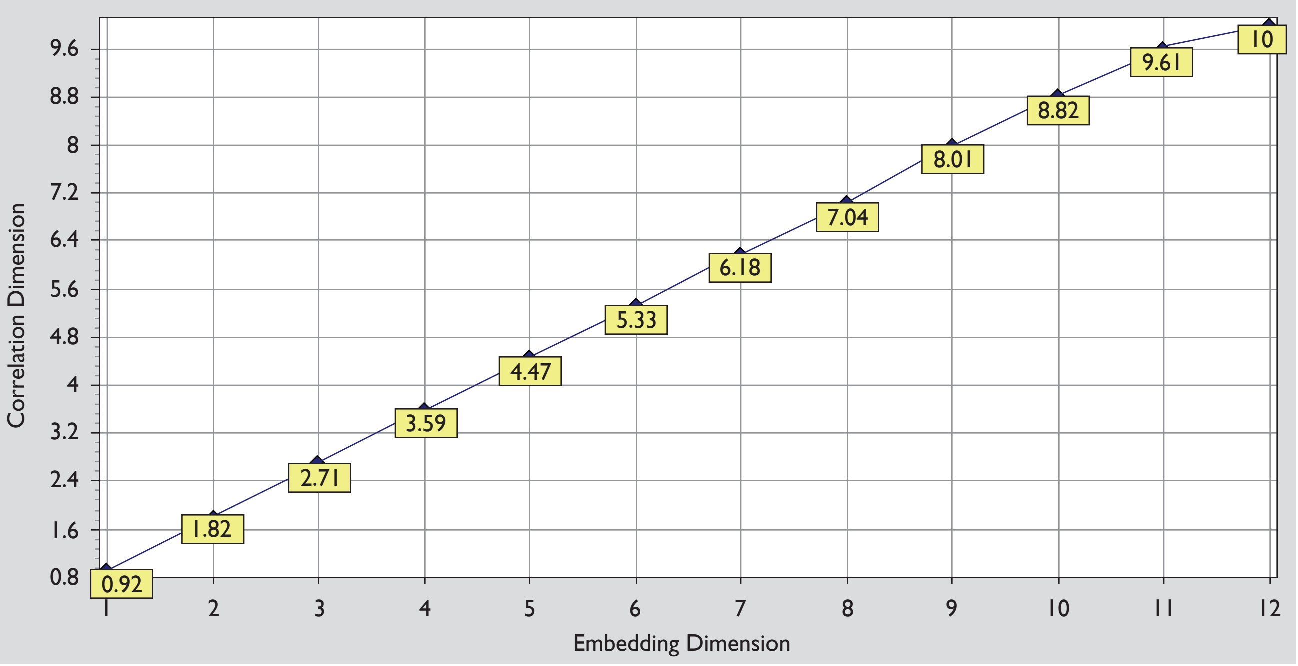

Figure 1(b). Plot of Correlation Dimension for Increasing Embedding Dimensions for AR-GARCH Residuals of NIFTY also be seen from this figure wherein it is clear that the correlation dimension plot of the filtered residuals of NIFTY returns does not coincide with that of the original plot. Thus the series fails Brock's residual test and this indicates the absence of deterministic chaos.

In sum, stock markets exhibit linear deterministic processes and are neither random nor chaotic. This has several interesting implications. First, the rejection of the efficient market hypothesis by the BDS test indicates that investors can gainfully employ a number of investment strategies which would usually fail in efficient markets. Second, since forecasting does not become increasingly unpredictable into the future (as would happen in case markets were chaotic), long term forecasting could be possible with the application of appropriate non-linear deterministic models. Finally, the market, when predictable, is vulnerable to manipulators: this could be a wake-up call for regulators who prefer to see the markets efficient. It can however be expected that as more and more trading models are applied to exploit these inefficiencies, the market will attain efficiency through the utilization of opportunities by investors.

Forex Market

The results of the tests on the Exchange Rate time series are presented in Table 2 (BDS test) and Figure 2 (Correlation Dimension test). The BDS test results indicate that the null hypothesis gets rejected for all dimensions, sugges-ting that the series is not independently and identically distributed. Subsequently, the results of BDS test on AR-GARCH residuals of exchange rate show that the null hypothesis gets rejected for all dimensions. In other words, AR-GARCH does not fit the exchange rate series adequately. This result suggests that the original series may be characterized by non-linear deterministic dependence, including chaos.

The correlation dimension plot for the exchange rate series (Figure 2(a)) shows that the correlation dimension values keep increasing with the embedding dimension but with a slope that is less than 1. From embedding dimension 10 to dimension 12, the correlation dimension values increase from 3.63 to 4.13. Further, from Brock's residual test the correlation dimension plot (Figure 2(b)) of the residuals does not coincide with the correlation dimension plot of the original series suggesting that it may not have a chaotic generator.

As in the stock market, the major implications of detecting a deterministic generator for the exchange rate series would be the host of opportunities it would offer to speculators, hedgers and arbitrageurs. Therefore, our results indicate the presence of potential gains for traders in currency spot and derivatives markets if the actual non-linear deterministic generator can be identified.

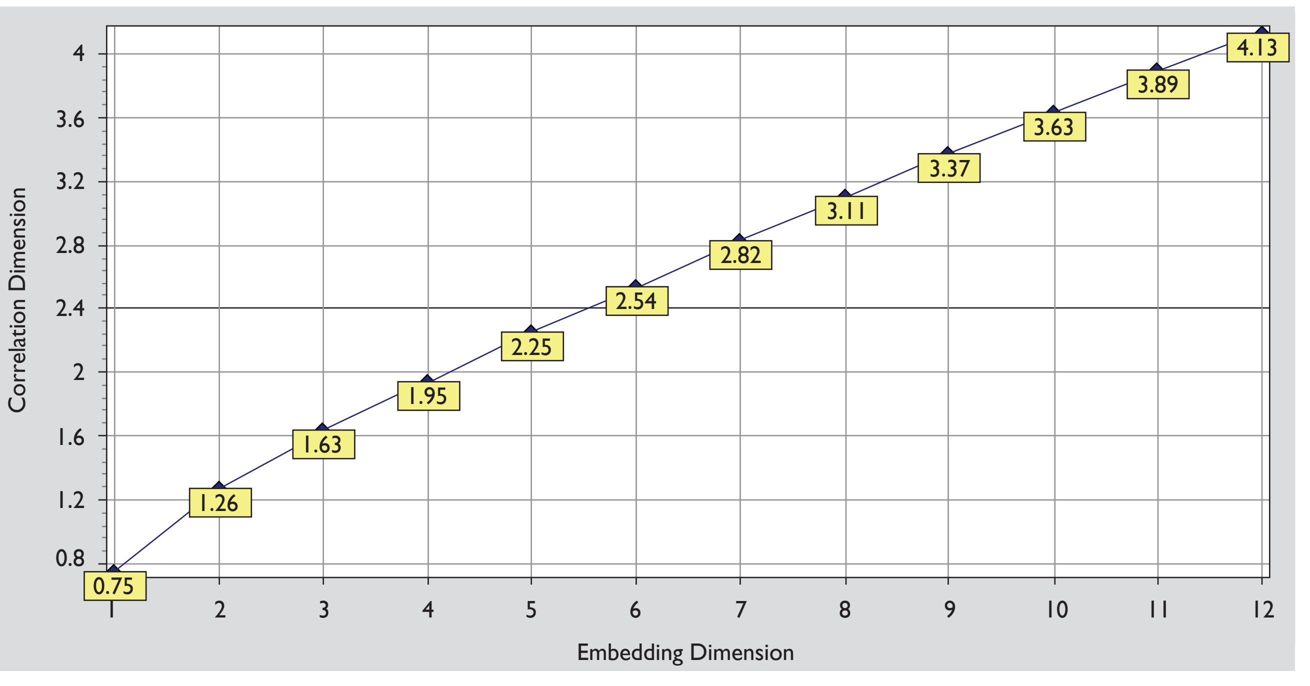

Figure 2(a). Plot of Correlation Dimension for Increasing Embedding Dimensions for Exchange Rate

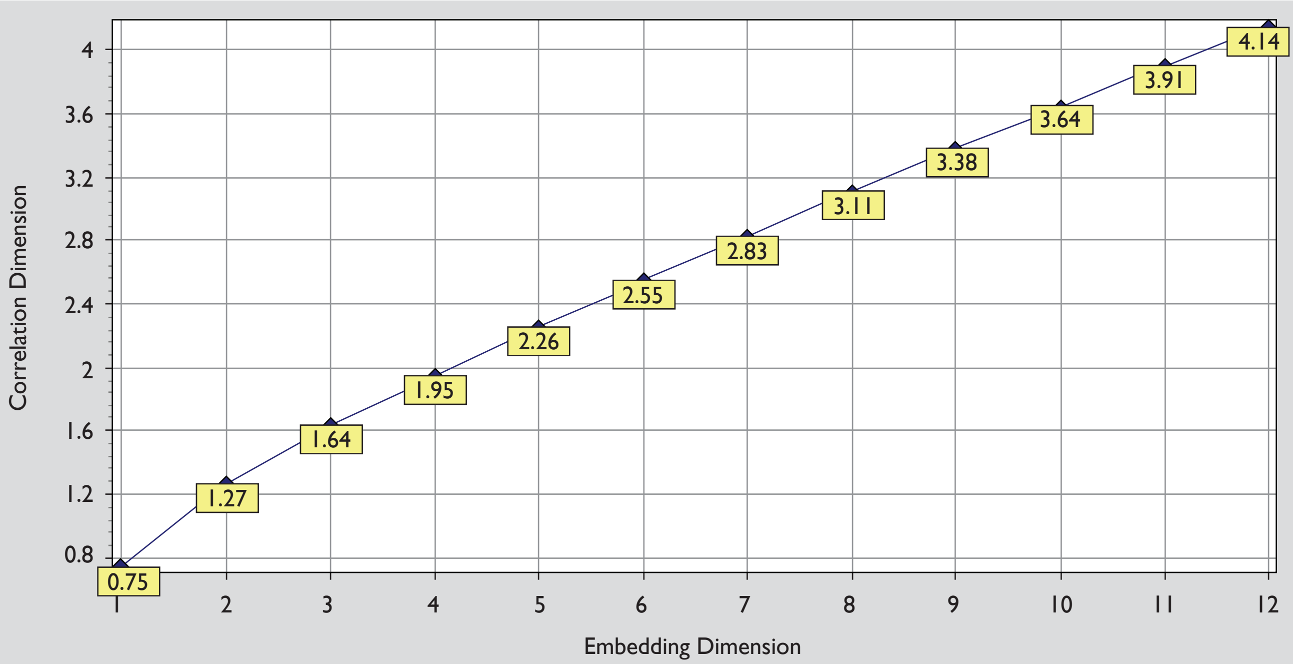

Figure 2(b). Plot of Correlation Dimension for Increasing Embedding Dimensions for AR-GARCH Residuals of Exchange Rate

The finding of deterministic behaviour implies that the spot rate can be predicted into the future and can be compared with available forward rates and depending on the differential, a profitable position can be taken.

Money Market

Table 2 and Figure 3 provide the results of the tests on the Call Money Rate series. From the BDS test results, it appears that the series is non-random. The result of BDS test on AR-GARCH residuals once again indicates that AR-GARCH is not an appropriate process to model this series. This suggests that the series may have a non-linear deterministic generator. The correlation dimension plot for the original series (Figure 3[a]) shows that, with increasing embedding dimension, the correlation dimension plateaus at around 3.14 indicating the possibility of chaos. However in the Brock's Residual test, the correlation dimension plot of the residuals (Figure 3[b]) does not coincide with the correlation dimension plot of the original series. This indicates that the series may not be chaotic. But since Brock's residual test is not a very powerful test especially with less than 4,000–5,000 data points (Trippi, 1995), it would not be prudent to outright reject the possibility of a weak chaotic generator for the series. Hence we do not recommend outright rejection of the possibility of a chaos in the money market. However the evidence is, at best, weak.

The implication of a deterministic call money rate series is potentially limited since it is rarely used for purposes of speculation or hedging—the main benefactors of predictability—and call money based derivative instruments are rare in India as in other emerging economies.

Bond Market

The results of the tests on the bond returns index are presented in Table 2 and Figure 4. The BDS test results suggest that the series is non-random. From the results of BDS test on AR-GARCH residuals, it appears that AR-GARCH fails to fully explain this series. Once again this may indicate the presence of a non-linear deterministic series. The correlation dimension plot for the original series (Figure 4[a]) reveals that the increase in correlation dimension slows down at around 2.38 indicating the possibility of chaos. However since the saturation or plateauing is not perfect, this result provides weak evidence of chaos. In the Brock's residual test, the plot of correlation dimension of the residual series (Figure 4(b)) clearly does not coincide with the plot of correlation dimension of the original series. This indicates that the bond returns data may not be chaotic.

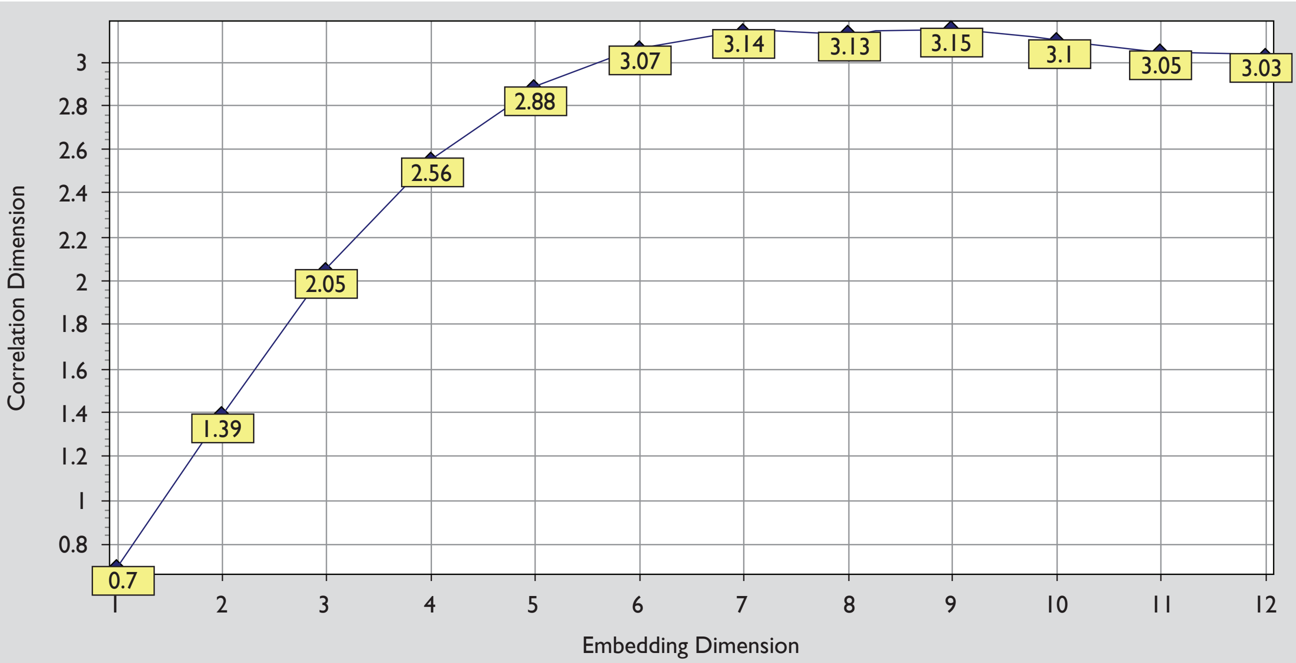

Figure 3(a). Plot of Correlation Dimension for Increasing Embedding Dimensions for Call Money Rate

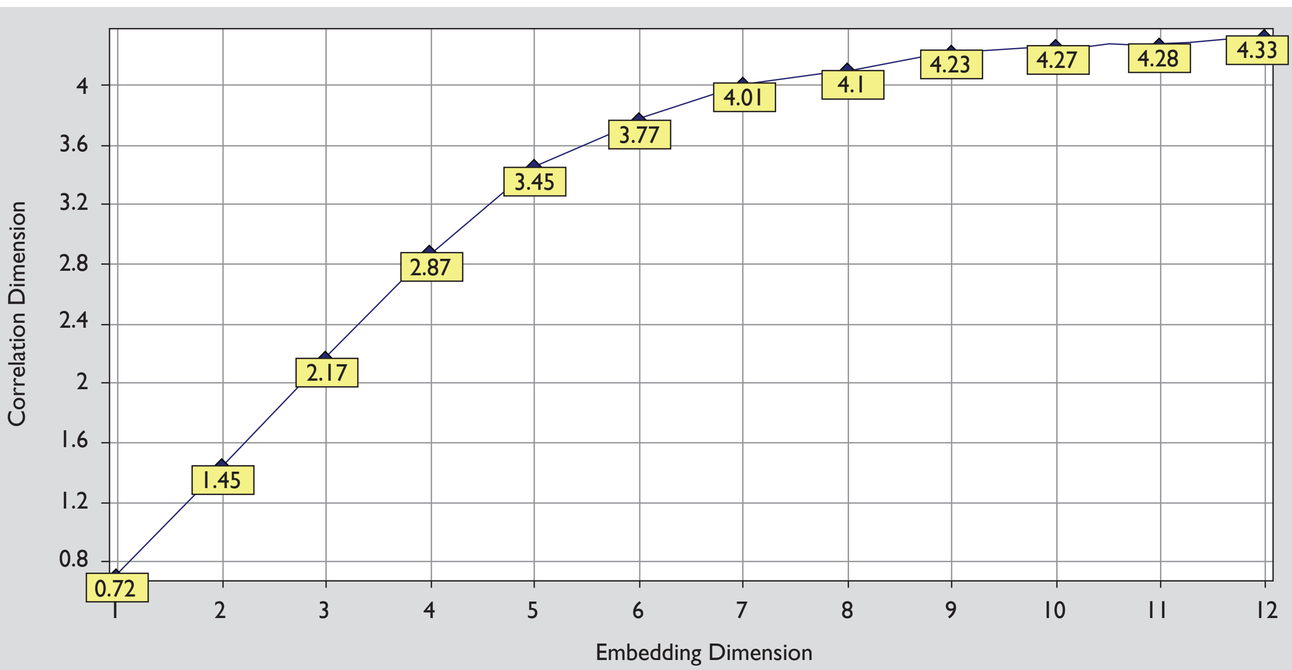

Figure 3(b). Plot of Correlation Dimension for Increasing Embedding Dimensions for AR-GARCH Residuals of Call Money Rate

Our findings for the bond market indicates that the deterministic long term yield rate series can be used by intelligent investors employing the appropriate non-linear deterministic models to take profitable positions in the bond market. Again, a possible chaotic generator (as suggested by the result of the Correlation Dimension test) would restrict the predictability of the models to limited time periods thus constraining their utility in the long term.

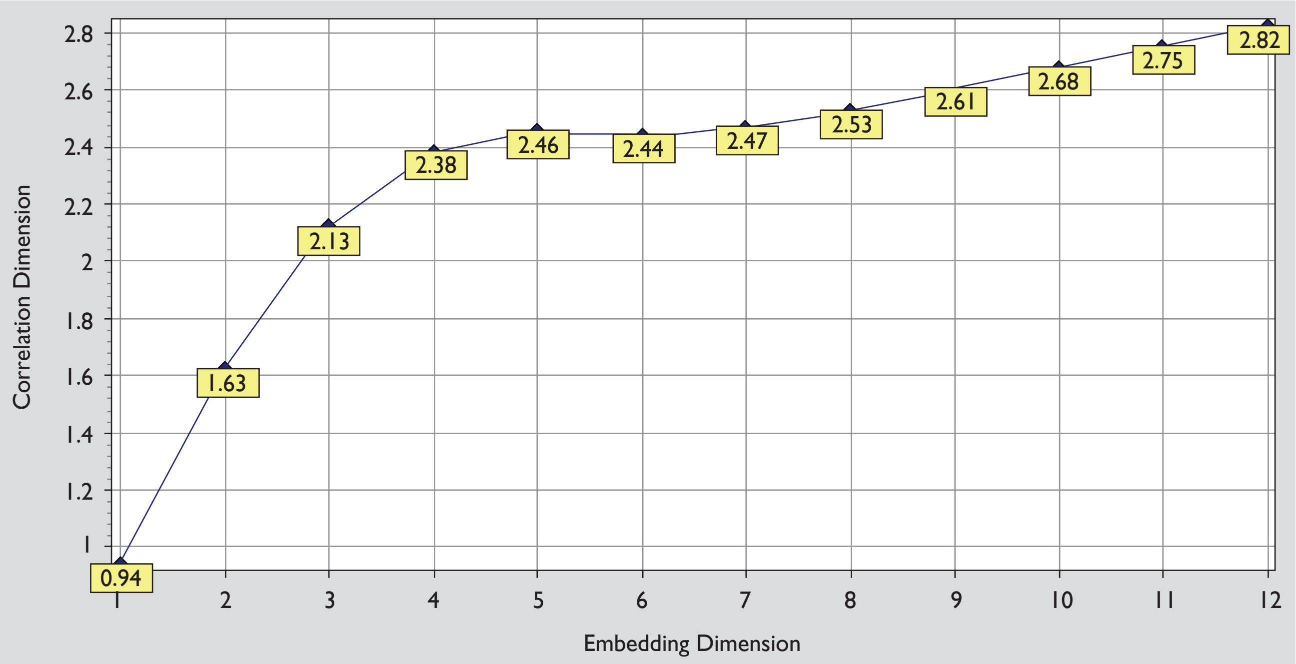

Figure 4(a). Plot of Correlation Dimension for Increasing Embedding Dimensions for Bond YTM

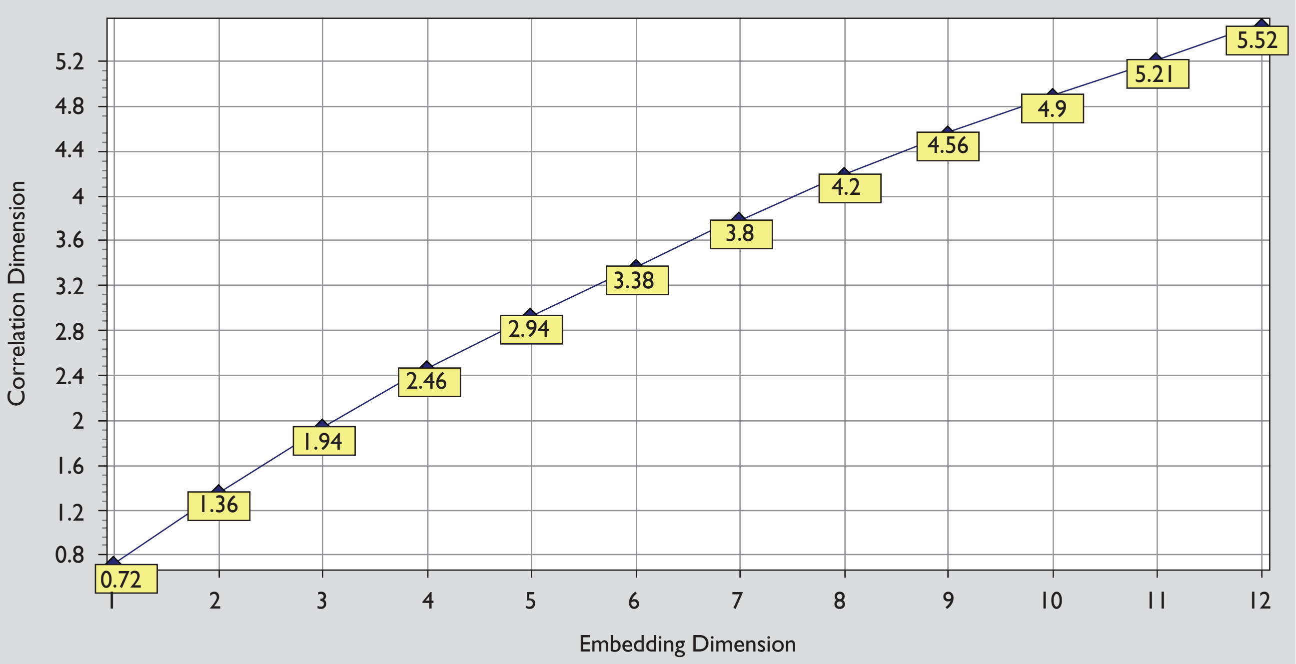

Figure 4(b). Plot of Correlation Dimension for Increasing Embedding Dimensions for AR-GARCH Residuals of Bond YTM

Conclusion and Implications

Chaos refers to a non-linear deterministic process that passes standard tests of randomness because such processes have moment properties that are identical to white noise. This article argues that detection of chaos is important in financial markets, since the presence of chaos implies that short term prediction (based on the non-linearity) is a possibility, provided that the true underlying generator is known. However while there are numerous empirical studies of non-linearity (including chaos) in developed financial markets, such studies for emerging financial markets are scarce.

In this article we conducted four tests of chaos on various financial markets of an emerging economy, such as India. Long time series of daily observations were collected on stock market indices, exchange rate, call money rate and bond returns. BDS test on the raw data and on pre-whitened data, Correlation Dimension test and Brock's residual test were conducted on each of these series. The findings of the paper can be summarized as follows. Our results provide evidence for non-randomness in each of the financial markets considered. We conclude that all these series are characterized by deterministic non-linear dependence, though not necessarily chaos. The lack of evidence for chaos in stock markets is consistent with the findings of Mishra *et al.* (2011). More generally our results indicate the rejection of efficient market hypothesis for Indian financial markets and point towards a scope for hedging and gains through speculation. Moreover, given the non-chaotic deterministic nature of these series it should be possible to forecast into the long term with minimal loss of predictability provided the appropriate non-linear deterministic generator is identified and employed.

Appendix 1. Statistical Tests Employed

Test-1: BDS Test on Raw Data

We use the BDS test (developed by Brock, Dechert, & Scheinkman, 1996) to test the null hypothesis of whiteness (randomness) using a non-parametric technique. The test statistic is given as

$$W(N,m,\varepsilon)=\sqrt{N}\,\frac{C(N,m,\varepsilon)-C(N,1,\varepsilon)^m}{\hat{\sigma}(N,m,\varepsilon)}$$

Where N is the sample size, $C(N, m, \varepsilon)$ represents the correlation function and $\hat{\sigma}(N,m,\varepsilon)$ is an estimate of the asymptotic standard deviation of $C(N, m, \varepsilon) - C(N, 1, \varepsilon)^m$. The correlation function $C(N, m, \varepsilon)$ gives the probability of the distance between any two m-histories, X_t and X_s, of the time series denoted by $\{x_t\}$ being less than a small number ε. If $\{x_t\}$ the time series were independent and identically distributed, then the joint probability would equal the product of the probabilities and all the probabilities would be equal. Accordingly, the BDS statistic allows us to test the null hypothesis that $C(N, m, \varepsilon) = C(N, 1, \varepsilon)^m$, which is equivalent to the null of randomness. As the asymptotic distribution of the BDS test statistic follows a standard normal distribution under the null hypothesis, the BDS test can provide a direct test of randomness or whiteness against general dependence, which includes linear and non-linear dependence.

Test-2: BDS Test on Filtered Series

This is a special application of the BDS test wherein a linear or non-linear stochastic model is first fitted into the original series and the residual series is subjected to a BDS test. Thus the BDS test on the residuals can be used to check if the best fit model for a given time series is a stochastic model failing which there is a possibility of deterministic non-linearity in the data. Since financial time-series are known to exhibit both dependence as well as volatility clustering, we use appropriate AR-GARCH models (based on information criteria) to filter the original series in all cases and then apply BDS test on standardized residuals.[3]

Test-3: The Correlation Dimension Test

Given a large number of observations, the proportion of data points with distances between them less than a pre-determined value serves as a measure of the randomness in data and this measure is called the correlation integral. Formally, the correlation integral $C(\varepsilon)$ for a time series is defined for different length scales ε, by the expression,

$$C(\varepsilon) = Lim_{n\to\infty}[1/N(N-1)]\sum_{i\neq j}^{N} H(\varepsilon, X_i, X_j)$$

where N is the sample size, X_i, X_j are observations in the time series and $H(\varepsilon, X_i, X_j)$ is the Heaviside function:

$$H(\varepsilon, X_i, X_j) = \begin{cases} 1 \; if\; |X_i - X_j| < \varepsilon \\ 0 \; if\; |X_i - X_j| > \varepsilon \end{cases}$$

Grassberger and Procaccia (1983) show that for small ε, $C(\varepsilon) = Constant \times \varepsilon^d$ where the exponent d is called the correlation dimension. Thus, correlation dimension for a particular embedding dimension[4] (call it k) is the slope coefficient estimated from the regression of $ln\ C(\varepsilon)$ on $ln\ (\varepsilon)$ for small ε.[5] To obtain an estimate of the true correlation dimension, the time series is embedded in successively higher dimensions till k converges to a stable value (indicated by plateauing of the graph of k) which is the true correlation dimension value. In case of white noise, as the number of embedding dimensions increases, the correlation dimension increases at the same rate throughout (i.e., slope of the graph is 1 and the correlation dimension is equal to the embedding dimension). If the correlation dimension increases, with an increase in the embedding dimension, but at a much slower rate (i.e., slope of the graph much lesser than 1), it suggests a deterministic system which is not chaotic. If the correlation dimension eventually stabilizes (also known as saturation) then the system is chaotic.

Test-4: Brock's Residual Test

Brock (1986) argued that the estimated correlation dimension of the residuals from the best fitting serial generator model must be the same as that of the original series if the data is chaotic. If the data are stochastic in nature, the dimension of the residuals will increase since they have less structure than the original data (Yang & Brorsen, 1993). The key is to first make the residuals as close to white noise as possible by filtering with traditional linear or non-linear stochastic models and then check the residual series for chaos. To confirm deterministic chaos, diagnostics should meet the saturation condition as well as Brock's residual test. That is, beyond some embedding dimension, estimated correlation dimension for both raw and residual data should be the same and also stabilize (Yang & Brorsen, 1993).

Acknowledgement

We are grateful to Saptarshi Purkayastha for his help.

Notes

1. Engle's LM test indicated the presence of ARCH effects in all the series considered.
2. The results are available on request.
3. In case of all the financial markets under consideration, an AR(1)-GARCH(1,1) specification gave the best fit, as indicated by various information criterions.
4. Embedding dimension or Euclidean dimension refers to the number of coordinates that is necessary to define a point.
5. While there can be different ways of choosing ε, we select ε to ensure that a certain fraction of the total number of pairs of points in the sample lie close to each other. This method is robust to different distributions of the underlying series.

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