How to Win Acceptance of the Inequality Process as Economics?

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Abstract

The Inequality Process (IP) is a particle system model similar to that of the Kinetic Theory of Gases. The IP is a parsimonious model of competition among people for wealth. The IP explains a wide scope of stable patterns in the distribution of personal income and wealth. Econophysicists have adopted the IP as part of their field, but the IP has been ignored or rejected by economists even though economists claim expertise on the distribution of personal income and wealth. The academic discipline of statistics in the US claims expertise on data analysis. Yet from the mid-twentieth century on, advances in computationally intensive algorithms for data analysis were developed largely outside of the discipline of statistics. Not until experts on this new paradigm of data analysis diverted resources away from traditional, old paradigm statisticians, was the new paradigm widely accepted in the discipline of statistics, even though a few statisticians had contributed to computationally intensive data analysis all along. This article's thesis is that the IP will follow a path into economics similar to that taken by computationally intensive data analysis into statistics, once useful applications of the IP are found and experts on the IP divert resources away from economists. That day is not at hand. There are no applications of the IP to business or government at present. One conceivable application of the IP to market research, small area estimation of personal income distribution, is suggested.

Keywords

Economics, econophysics, income distribution, inequality, paradigm, particle system, scientific revolutions

Introduction

The Inequality Process (Angle, 1983–2012) is a stochastic particle system model of personal income and wealth statistics. The Inequality Process (IP) is similar to the stochastic particle system model of the Kinetic Theory of Gases (KTG), the mechanical basis of gas thermodynamics (Angle, 1990). Although published as mathematical sociology, the Inequality Process (IP) has been adopted as econophysics by econophysicists (see Appendix 1). Indian physicists have played a key role in this adoption, in particular Bikas K. Chakrabarti of the Saha Institute of Nuclear Physics and his associates, the organizers of Econophys-Kolkata, an international, biennial conference on econophysics. Econophys-Kolkata shows that India is a world centre of econophysical research.

This special issue of IIM Kozhikode's Society and Management Review on econophysics is far-sighted in its anticipation of the rewards to be eventually reaped from econophysics as it either extends or replaces parts of today's academic discipline of economics. While as yet there are few applications of econophysics outside of finance, it is conceivable there might be a 'first mover advantage' to a firm's applying a new econophysical law before the competition. The present article develops the thesis that economists will adopt the Inequality Process only after there are profitable applications of the Inequality Process. The present article suggests a possible first application of the Inequality Process (IP), perhaps of interest to market researchers. This application of the IP is 'small area estimation' of the distribution of personal income, the estimation of incomes in an area too small for there to be government statistics on the incomes of people residing there.

The Inequality Process (IP)

The Inequality Process (IP) explains a wide scope of personal income and wealth phenomena. It is possibly a natural statistical law similar to statistical laws of thermodynamics, universal and pervasive in all populations. Much more empirical testing of the IP than has been done is necessary before the IP can be acknowledged as a

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scientific law. However, it has already been shown with data from the US that the IP explains with parsimony, scope, and precision many features of personal income distributions and their scalar statistics (for example, statistics thought to be indicative of the concept 'inequality'). See Appendix 2 for the demonstrated empirical scope of the IP. For example, the IP accounts *inter alia* for the US distribution of annual wage and salary income conditioned on a worker's level of education (see Figure 1). The dotted piecewise linear curves in Figure 1 are the fitted IP estimates. The IP also jointly puts a number of familiar verbal propositions, each conventionally asserted by mainstream economists without recognition that these propositions are linked by the Inequality Process, on a firm mathematical and empirical footing for the first time, see Appendix 3.

The Inequality Process (IP) models a competition process for wealth in a population of particles. Each of these particles is an extremely simplified representation of a person, which is why the entities of the population are called 'particles' rather than 'people'. 'Particle' emphasizes the extreme parsimony of the model. The IP transfers wealth from particles that by the IP's meta-theory and by empirical referent are less productive of wealth to those more productive of it. While the best evidence that the Inequality Process pervades a whole national population (the US) is quantitative, the best evidence for the IP's universality is qualitative: its accounting for the distribution of wealth in cultures documented by anthropologists, historians, or sociologists throughout time, space, and techno-cultural evolution. An example of a widely documented, qualitative fact that the IP accounts for is the universal pairing of the earliest evidence of great concentration of wealth in the same archaeological strata as the earliest evidence of an abundance of stored food, the subject of the 'surplus theory of social stratification' in economic anthropology.

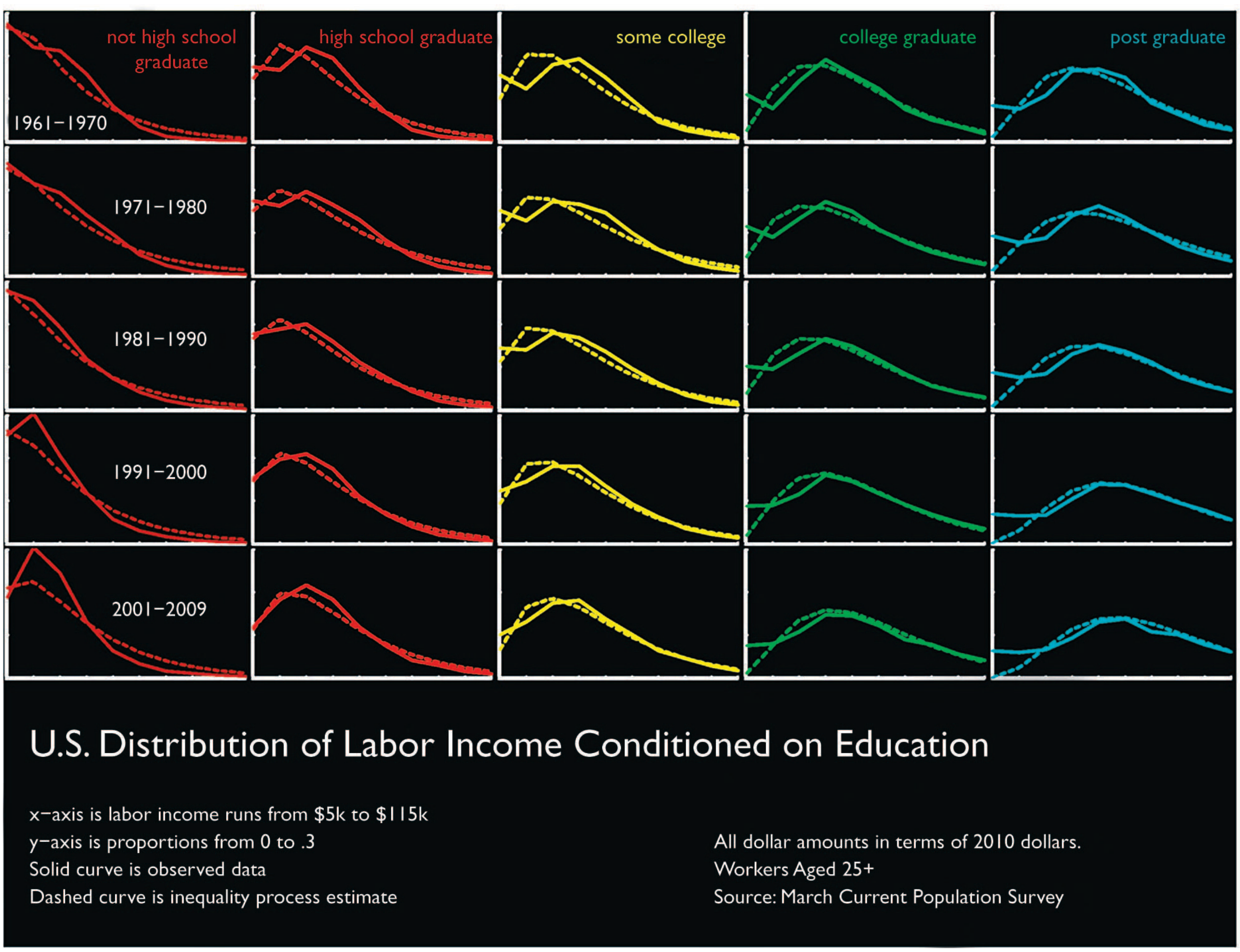

Figure 1.

The Inequality Process (IP) is a conservative interacting particle system in which particles are randomly paired to compete for each other's wealth. A winner is chosen randomly, that is, winning or losing is not dependent on particle characteristics and in the long run, each particle wins and loses 50 per cent of all competitions. The fraction of wealth the losing particle gives up to the winning particle in a loss is an unchanging characteristic of each particle. 'Conservative' means that the positive quantity, called 'wealth', exchanged between particles is neither created nor destroyed. It is a simplifying assumption, like the isolation and immortality of the population of particles. The population of particles is partitioned into equivalence classes by the particles' parameter, the fraction of its wealth it gives up when it loses. In the IP, wealth is transferred to robust losers, those that lose less when they lose. See Appendix 4 for the equations governing the transfer of wealth between particles in a competitive encounter, the IP's transition equations. It is these equations that distinguish the Inequality Process (IP) from the Kinetic Theory of Gases (KTG) and from the more closely related model of Chakraborti and Chakrabarti (2000).

Criticisms Some Economists Have Directed Toward the Inequality Process (IP) and Similar Models

Verisimilitude

Despite the evidence that the Inequality Process (IP) is a pervasive, universal competition process whose statistical signature is all over personal wealth and income, most economists who have learned about the IP judge it to be 'not economics' regardless of its merits as science or mathematical sociology. Since an economist has yet to challenge the validity of the IP as science, the judgement that the IP is 'not economics' is most reasonably interpreted as the proposition that the IP is, in Thomas Kuhn's term in *The Structure of Scientific Revolutions* (1992), 'incommensurate' with economics, that is the IP does not fit into the established paradigm of economics. Alternatively, one might say the IP lacks verisimilitude as economics for economists. Prof. Thomas Lux of the Department of Economics of the University of Kiel, Germany, called the Inequality Process to the attention of the first EconoPhys-Kolkata conference (Lux, 2005), announcing the IP's priority as a particle system of wealth and income. At the same time, Lux describes the IP as something other than economics. Lux labelled the IP a 'toy' model, one that demonstrates a principle—stochastic effects on personal income and wealth—but one that is too simple to be relevant to the economics of an industrial economy, despite the IP's quantitatively implying many stable empirical patterns in the US income and wealth statistics. See Appendix 2.

Verisimilitude is a lagging attribute of a successful model. Witness the Kinetic Theory of Gases (KTG), proposed by Daniel Bernoulli in 1738 (Stillwell, 1989) when there was no evidence for the atomic theory of matter. The success of the KTG as an explanation for gas thermodynamics gave verisimilitude to the atomic theory of matter, not vice versa. Angle (1990) describes how closely the IP resembles the KTG. The IP shares the KTG's parsimony. A parsimonious model is not necessarily a toy because it is parsimonious. Brodbeck's (1959) example of a model with verisimilitude is a miniature replica of a railroad. A model railroad is a toy. Verisimilitude is a virtue in simulations whose realism entertains and/or instructs (for example, model railroad or flight simulator).

Generalized Parsimony over Verisimilitude

Parsimony, narrowly defined as model simplicity, is, in itself no guarantee of relevance in the search for new natural law or even a signpost toward discovery. Generalized parsimony, however, announces a candidate for scientific law. Generalized parsimony combines the properties of (a) model simplicity, (b) internal consistency, (c) wide empirical explanatory scope, and (d) disconfirmability (that is, sufficient rigidity and specificity of implications so that the model can be disconfirmed via logical or empirical test (cf. Popper, 2000). These elements of generalized parsimony are similar to a subset of elements in Kuhn's (1992) set of attractive elements in a scientific law. The specification and testing of the Inequality Process is intended to enhance its generalized parsimony:

Verbal theory assigning meaning to variables → abstraction as mathematical model trying to max simplicity → derivation of hypothesis → test of hypothesis against data → empirical confirmation of hypothesis → derivation of different hypothesis → test against data → empirical confirmation → and so on, widening the scope of empirical phenomena explained by model → inductive establishment of Inequality Process as scientific law.

Appendix 4 describes the meta-theory from which the IP was specified. Recognition of the importance of generalized parsimony is rare in sociology and economics. On the contrary, in the author's experience, the great majority of social scientists see a mathematical model with generalized parsimony as severely deficient in verisimilitude. In this regard Lux (2005) and Gallegati, Keen, Lux, and Ormerod (2006) are mainstream. Verisimilitude in the social sciences in the US is achieved by engagement with disciplinary icons, relevance to the government and news media concerns of the day, staying within the discipline's established paradigm (in the sense of Thomas Kuhn's *The Structure of Scientific Revolutions*), and scholarly ancestor worship.

The greatest verisimilitude problem that Prof. Lux and three other economists identify in the IP and similar particle system models of income and wealth (Gallegati *et al.*, 2006) is the elision in these particle systems of the distinction between the flow (income) and stock (assets) concepts of wealth. Particle wealth in the IP is a stock concept and the majority of the IP's quantitative tests and confirmations are against personal income data, particularly personal income from labour. Most of the stock of wealth in modern industrial countries is in the form of human capital, mostly people's educations, as can be ascertained by estimating the quantity of tangible assets required under extant interest rates to generate an income equivalent to that from labour. In contemporary economies, human capital's share of national wealth is greater than that of tangible capital and natural resources combined (Hamilton & Liu, 2013; Jorgenson & Fraumeni, 1989). Consequently in such economies, personal income (most of which is labour income, the rent on human capital) is the best measure of the stock of wealth. If one only thinks in terms of conformity to current economic theory, then Gallegati *et al.*'s (2006) vigorous rejection of the class of particle systems that includes the IP makes some sense, but not if one prizes generalized parsimony, as physicists do.

The Inequality Process (IP) Has No Dynamics?

Prof. Lux's criticisms of the IP and other particle systems of wealth and income go beyond its lack of verisimilitude as economics. Prof. Lux's principal critique of the IP in particular (Lux, 2005) and the class of particle systems that includes the IP in general (Gallegati *et al.*, 2006) is that the IP and others in its class (e.g., Chakraborti & Chakrabarti, 2000; Dragulescu & Yakovenko, 2000) are incapable of modelling a modern economy because a modern economy is dynamic. By 'dynamic' Gallegati *et al*. (2005) mean that economics is about growth in economic product. They infer that the IP and similar models are incapable of modelling economic growth because they are 'conservative', in the following sense: transfers of wealth between particles neither create nor destroy wealth. In a simple version of the IP the population of particles is immortal, the size of the population and total wealth are constant. So, mean wealth does not change. Are Lux (2005) and Gallegati *et al.* (2006) right to conclude from these facts that the IP and similar models are irrelevant to modern economies because the IP and other conservative particle systems have no dynamics?

The short answer (Angle, 2006e) is that if Gallegati *et al.*'s (2006) critique of conservative particle systems as incapable of dynamics were true, then the Kinetic Theory of Gases (KTG), a conservative particle system, would not be the micro-level basis of gas thermo*dynamics*.

A more detailed answer is that as long as the empirical process the IP models converges to its stationary distribution faster than the aggregate total of wealth changes, the aggregate total of wealth can be treated as an exogenous variable by the IP, a possible exogenous driver of its dynamics. The Macro Model of the Inequality Process (MMIP) is a functional form derived from the solution of the IP's transition equations (see Appendix 4). The MMIP approximates the stationary distribution of the IP in terms of its parameters. It was developed over a chain of papers (Angle, 1992, 1999a,b, 2002a-c, 2003a,c, 2005, 2006a,b, 2007a). The MMIP treats the unconditional mean of the empirical income or wealth distribution it fits as an exogenous variable, one only appearing in the MMIP's scale parameter. This unconditional mean is estimated by an approximation formula for the MMIP's median. The MMIP fits time-series of personal income distributions and implies fits to their scalar statistics in the last half century of data from the US.

Tenacity of Belief in the Traditional Paradigm of Economics

People with PhDs in physics have shared in the income and prestige of quantitative financial specialists (Overbye, 2009). Finance is more pragmatic than the social sciences. The validity of a financial model is immediately apparent in its profitability; underperforming models are abandoned. Results of testing sociological or economic theories or models are usually not as clear as in finance. There is a tenacity of belief in the social sciences that sets up cognitive dissonance, that is, a predisposition to perceive and remember confirmation of prior belief, and vice versa for anything not fitting prior belief. Deeply rooted elements of ancient cultures are embedded in economics. Economics contains elements of ancient wisdom about how to organize one's life: hard work, saving, and investment (e.g., Aesop's *Fable of the Ant and the Grasshopper*). Elements of economics have been associated with religion, a culturally conservative institution (e.g., Max Weber's *The Protestant Reformation and the Spirit of Capitalism*). Indeed, in the US, neoclassical micro-economics itself has been referred to as the 'old time religion'. 'Old time' in this context has meanings like 'inherited unchanged', 'unquestioned', and 'fundamentalist'. Gallegati *et al.* (2006) specifically warn physicists away from particle system models of income and wealth because particle systems of personal income and wealth distribution have, unlike purely financial models in their view, substantive economic implications. Gallegati *et al.* (2006) are correct on this point. Via a footnote to Lux (2005) on the Inequality Process, Gallegati *et al.* (2006) assert that particle systems of wealth and income may be acceptable as anthropology but not as economics. This assertion seems like the statement that conservative particle systems are taboo as economics. Even though, as Appendix 3 shows, the IP makes familiar economic propositions joint implications of a mathematical model with generalized parsimony, the IP may be seen as an unwelcome alien intruder, certainly an upstart.

A Personal Memoir of a Smooth Paradigm Shift Facilitated by the Use-Validity of a New Paradigm

The Arrival of Statistical Learning, A New Paradigm in Statistics

The author witnessed in the last several decades of the twentieth century a paradigm shift in a science, statistics, also known as 'mathematical statistics', that was, by the standards of the slow, rancorous paradigm shifts chronicled by Thomas Kuhn in *The Structure of Scientific Revolutions*, remarkably swift and quiet. The new paradigm in statistics goes by a variety of names: 'knowledge discovery', 'machine learning', 'data mining', 'data science', 'data analytics', 'business analytics', 'predictive analytics' or just 'analytics'. Also 'Big Data' has also been used to refer to the new paradigm since the new paradigm may be particularly useful with big datasets. After the discipline of statistics had accepted the new paradigm it gave the new paradigm yet another name, 'statistical learning'. 'Statistical learning', the new paradigm's new name in statistics, is of recent vintage, possibly popularized by the title of Hastie, Tibshirani, and Friedman's (2001) book, *Elements of Statisical Learning*. Experts may draw distinctions among the various names for the new paradigm, but most people new to the statistical learning paradigm focus on the commonalities. The new paradigm is data analysis via computationally intensive algorithms able to do data analysis more ambitiously than old paradigm data analysis. The algorithms of the new paradigm do things that the old paradigm of statistics had little interest in doing, or even frowned on, such as the ransacking, also called mining, of large databases for interesting information. The new paradigm has a variety of names because it is the product of researchers from a variety of disciplines. Some statisticians have participated in the new paradigm all along but they were few in number in the early days. Most of the new paradigm comes from the discipline of computer science. Although the new paradigm has, as of the turn of the twenty-first century, become represented in most US universities' departments of statistics, it encountered xenophobia, the 'not invented here' or the 'not what I was trained to do and not what I am interested in' response from many senior faculty who rule departments of statistics and define the discipline of statistics.

Disdain for Data Analysis in the Old Paradigm

The old paradigm in statistics is what statisticians were doing prior to the mid-twentieth century. Although under the old paradigm, statisticians claimed the subject of data analysis as their domain of expertise, data analysis was a low status activity for statisticians. It was tedious with the computational devices of the time. Consequently, samples were kept small to reduce computational load. While nearly all old paradigm statisticians did data analysis at one time or another, because of the service role of the discipline as steward of the decision to pronounce some results from small samples acceptable under probability theory (to pronounce those results 'statistically significant'), the mathematics of this decision and related theory were the frontier of the field.

In small samples the decision to reject the null hypothesis that the value of a test statistic is not statistically significant confounds sample size with the magnitude of the test statistic. The smaller the sample, the bigger the test statistic has to be to be pronounced statistically significant, and vice versa. The computer revolution of the mid-twentieth century led to the creation of huge databases as it led to software to relieve people of tedious computation. In 2001 the author heard a database manager of a US Internet marketing firm refer to a 17 exabyte sample as a toy sample. Given standard assumptions about how that sample was drawn, chances are that the issue of statistical significance, traditionally defined, hardly arose in its analysis or the analysis of larger samples, although the identification of unimportant, but statistically significant, noise in the data probably did. Nevertheless, where sample sizes are small because of high cost or ethical and time constraints, for example, pharmaceutical testing, the old paradigm has lost none of its relevance. Perhaps one reason for the relatively smooth acceptance of statistical learning into the core of the discipline of statistics is that the new paradigm, statistical learning, contradicts nothing in the old paradigm however much the new paradigm may force old paradigm loyalists to yield some pride of place, employment opportunities, research funding, and other resources. The new and old paradigms in statistics are complementary.

As a counter-example to the proposition that data analysis was disdained as a menial task in statistics prior to the computer revolution of the mid-twentieth century, one might point to Sir Ronald Fisher, one of the greatest statisticians of the first half of the twentieth century. Fisher was indeed involved with data analysis, but more so than the great majority of his peers in statistics, since he had a dual career. Fisher was also a scientist, a leading biologist. In the mid-1980s the author found out personally how little importance the department of statistics of a US research university attached to data analysis in the training of statisticians when a co-worker in the Research Division, National Office, US Internal Revenue Service in Washington, DC, who had a Master's in Statistics needed help with an elementary data analysis task. She complained that the curriculum of her master's programme ignored data analysis. Her master's programme had been in the advanced calculus needed for mathematical probability theory. She said that in the last week of the programme, an instructor told those not going on to the doctoral programme that a particular integral is a mean and another integral a variance and that, equipped with this knowledge, they were prepared to go forth and become data analysts.

For the author the paradigm shift to statistical learning began with Morgan and Sonquist's (1963), 'automatic interaction detector' (AID). The AID algorithm is now more commonly called 'classification and regression tree' (CART) analysis, as formalized in Breiman, Friedman, Olshen, and Stone (1984). It was during a lecture in the late-1980s on CART that the author learned that influential statisticians in the US looked askance at CART and related algorithms. The lecturer on CART used the first 10 minutes of audience attention to complain of bad treatment of statistical learning papers by major US statistics journals.

The Acceptance of the Statistical Learning Paradigm in Statistics

Soon after the turn of the twenty-first century the American Statistical Association (ASA) signalled to its membership that the statistical learning paradigm should be accepted. The ASA is the organizer of the Joint Statistical Meetings (JSM), a meeting of a number of statistical societies including the International Indian Statistical Association. JSM 2001 signalled to statisticians that the new paradigm, which the ASA referred to as 'data mining', was now part of the discipline by offering a number of sessions on data mining for statisticians trained in the old paradigm. Session 19, 'Teaching Data Mining', of JSM2001 was a panel session in a big room. What makes this session a particularly clear marker of the acceptance of the new paradigm in statistics is that, in the statistics jargon extant in the US in the 1970s and earlier, 'data mining' had meant the deceptive practice of testing and rejecting the null hypothesis that a test statistic is not statistically significant after selecting a statistic, previously known to be large, for the test of statistical significance. JSM2001 was not promoting that deception. Rather the meaning of 'data mining' in statistics by 2001 was entirely different from what it had been a little more than two decades earlier.

The new meaning of 'data mining' for statisticians came from computer science. Acquisition and storage of data has a cost. Computer scientists, responding to the desire of database managers to amortize that cost more quickly, designed algorithms to search databases for information that would reduce a firm's costs or increase its revenues. They called this activity 'data mining'. The fact that for statisticians the meaning of 'data mining' in computer science had replaced the meaning of 'data mining' in statistics in a little over two decades indicates that the new paradigm of statistical learning had overwhelmed resistance to it. The new paradigm's victory was recent in 2001. Even the name that statisticians would eventually use to refer to the new paradigm, 'statistical learning', was not the name the ASA used to schedule seminars on the new paradigm in

2001. The phrase 'statistical learning' appears in the title of Hastie, Tibshirani, and Friedman's (2001) *The Elements of Statistical Learning* but it was so new in 2001 that the authors had to explain what 'statistical learning' means in the subtitle, 'Data Mining, Inference, and Prediction'.

The audience of JSM2001's Session 19, 'Teaching Data Mining', filled the room and was more enthusiastic than any the author has seen at a JSM session. Session 19 was intended to help PhD statisticians trained in the old paradigm of statistics to master the new paradigm well enough to teach it. The organizer of the session stoked audience enthusiasm by mentioning that re-styling oneself as a data miner might enable PhD's in statistics to earn a salary that, for most, would be about a tripling. Another Session 19 speaker was a holder of an old paradigm PhD in statistics who had transitioned to teaching the new paradigm at a business school. He mentioned that students at his school viewed data mining as a subject entirely different from and fundamentally better than statistics. In some US business schools separate programmes or even departments had sprung up to teach the new computationally intensive algorithms for data analysis with names such as 'business analytics', 'predictive analytics', 'analytics', 'data mining', 'data science'… and so on. This speaker said much of the enthusiasm for the new paradigm of data analysis among students is due to industry demand. Session 19's panel offered a carrot to traditional departments of statistics, a large new source of funds, and a stick, desertion of students to programmes and departments teaching the new paradigm.

The American Statistical Association's (ASA) encouragement of computationally intensive data analysis is ongoing. Besides the sessions on data mining at JSM2001, the theme of JSM 2010 was 'Statistics: A Key to Innovation in a Data-Centric World', and that of JSM2012, 'Statistics: Growing to Serve a Data-Dependent Society'. ASA president, Prof. Marie Davidian, gave further impetus to the statistical learning paradigm in the July 2013 issue of the ASA's monthly news magazine, *The Amstat News*. In an article entitled 'Aren't We Data Science?' she expresses dismay that few statisticians and statistical departments were asked to join an organization promoting data science as an economic development strategy for a US state. She wrote of her concern that the discipline's claim to expertise on data analysis has been narrowed. She writes 'I've been told of university administrators who have stated their perceptions that statistics is relevant only to "small data" and "traditional" "tools" for their analysis, while data science is focused on Big Data, Big Questions, and innovative new methods.' The intention of the article is to refocus the discipline on the new paradigm of statistical learning (Davidian, 2013).

Will the Inequality Process (IP) Follow a Similar Path into Economics?

In the last decades of the twentieth century, the discipline of statistics in the US faced a situation in which its claim to expertise about data analysis was narrowed because a frontier in data analysis was largely pioneered by researchers from other disciplines, computer science in particular. Although not widely acknowledged, the Inequality Process (IP) has narrowed the claim of economics to expertise about stable patterns and trends in statistics of personal income and wealth. The IP is a mathematical model that jointly and quantitatively explains a wide scope of phenomena related to personal income and wealth. There is qualitative evidence pointing to the IP's universality in all populations, up and down the trajectory of techno-cultural evolution. The IP has generalized parsimony. See Appendix 2. There is no comparable model in economics. Can the IP win acceptance as economics by economists by following a path similar to that taken by the statistical learning paradigm into the academic discipline of statistics in the US?

In the light of the relatively quick and smooth acceptance of the new paradigm of statistical learning in statistics, facilitated by demand for the new paradigm by business, industry and government, perhaps the most likely answer to the question, 'How to win acceptance of the Inequality Process as economics?', would be to create demand for applications of the Inequality Process (IP). There is but one application of the Inequality Process to a task that might interest business, industry, or government that has gotten as far as a preliminary investigation. That application is the estimation of personal income distribution in a small area. Angle and Land (2010) apply the Inequality Process (IP) to this task in the case of two small suburbs adjacent to the City of Philadelphia in the US state of Pennsylvania. This demonstration is on so small a scale and so localized that it provides little assurance of the method working satisfactorily elsewhere.

Estimation of income or wealth distributions in a small area is necessitated by the absence of data. The smaller the number of people in a small area, the more reluctant a government statistical office is to publish data on that small area, particularly income and wealth data. Consequently, consumer market researchers tasked with estimating the distribution of personal income in a small area around a potential site for a store may need to do a small area estimation of personal income distribution to assess the potential local market for the store. The Macro Model of the Inequality Process (MMIP) would help in this task. Similarly, should a government cease publication of tabulations of personal income and wealth distributions or cease electronic publication of micro-data samples (individual survey records that preserve respondent anonymity) with personal income and wealth data, as some US legislators have urged the US government to do, MMIP estimates of personal income and wealth distributions would become the next best thing to data. Although it has never been tried, to the author's knowledge, the MMIP might be reliably estimated with data on consumers contained in commercial databases.

Small Area Estimation via the Macro Model of the Inequality Process (MMIP)

The phrase 'borrowing strength' is used in empirical Bayes estimation of small area statistics to refer to the use of information from adjoining, nearby, or enveloping areas to estimate a statistic of interest in the small area in question (Carlin & Louis 2009; Fay & Herriot, 1979). If 'borrowing strength' works, it does so because the process generating the unknown value of the statistic in the small area of interest is the same process generating its known values in adjoining, nearby, or enveloping areas. While that process is typically unknown, Angle and Land (2010) hypothesize that in the case of personal income distribution, that process is the Inequality Process. A valid statistical law, if applicable, is the most concentrated form of 'strength' for small area estimation.

The specific form the Inequality Process (IP) takes as the estimator of an income or wealth distribution in a small area is the Macro Model of the Inequality Process (MMIP), a mathematical expression that approximates the stationary distribution of the Inequality Process in terms of the IP's particle parameter. Each IP particle has a parameter, ω, omega, the fraction of the wealth it loses when it loses a competitive encounter with another particle. The meta-theory of the Inequality Process associates a smaller fraction of wealth lost in a competitive encounter with greater worker skill and productivity (conventionally measured by a worker's level of education), see Appendix 4. The IP treats ω as a semi-permanent characteristic of a particle since worker skill level endures through time. The harmonic mean of different particles' ω is denoted $\tilde{\omega}_t$. It has a time-subscript because skill levels in a labour force change over time. See Appendix 4 for the MMIP's equations.

Angle (2012) shows that the IP fits the right tails of US annual distributions of wage and salary income conditioned on education better than a similar particle system model that more closely resembles the KTG. The MMIP provides good fits to, in particular, the right tails, the relative frequencies of large incomes. The MMIP may be especially useful in the estimation of the right tails of income distributions (frequencies of people with large incomes) in small areas. Angle (1996) demonstrated the near invariance of the US distribution of labour income conditioned on education under geographic disaggregation from the national distribution to distributions in contiguous areas of about 100,000 people (Public Use Microdata Areas [PUMA's] of the US Bureau of the Census). This finding implies that the IP can estimate labour incomes in small areas in the US given information on the distribution of education of the workers residing in the area. If a summary statistic of their labour income, such as the median, is available, this statistic becomes a constraint on the small area estimate of personal income distribution.

Conclusions

In recent centuries the discovery of a scientific law typically precedes practical applications to business, industry, or government. IP is a candidate for acknowledgement as scientific law, a statistical law similar to those in, for example, thermodynamics (Angle, 2011). The IP may be more than a descriptive law of income and wealth distribution and related phenomena. An initial guess at answers to the questions of why the IP works well as a model of such phenomena, why it appears to pervade a whole national population, and why it appears up and down the trajectory of techno-cultural evolution is that there is a single answer to all three questions: the IP is a fundamental economic process in all groupings of people, a competitive process whose statistical signature may be on many more phenomena than those it has been found on to date. Angle (2002a) speculates that the IP is the human analogue of the competition process that population biologists think allocates resources to individuals of all species, a process that enlarges species niche and maximizes population size. If the IP is that fundamental, it might imply new economic laws and strategies, yielding profits to the 'first movers' who exploit them.

Crowd-sourcing the Inequality Process' (IP's) First Use-Validation

Finding quantitative evidence of the IP's universality inductively in country after country would strengthen the IP's claim to generalized parsimony and standing as a scientific theory. However, given Thomas Kuhn's examples in *The Structure of Scientific Revolutions* of influential people in scientific disciplines defending their life's work, their status as experts, their prestige and income by defending an old paradigm, one might expect stout, perhaps even rancorous, opposition to the IP from economists particularly in the light of how embedded deeply held belief systems are in the economic paradigm. Indeed, Gallegati *et al.* (2006) may be a small foretaste of what is to come. Gallegati *et al.* (2006) was sufficiently confrontational to be covered as science news in a four-page feature article in *Nature* (Ball, 2006).

The quiet and smooth adoption of the 'statistical learning' paradigm into the core of the discipline of statistics in the US probably occurred because (a) the new paradigm invalidated nothing in the old paradigm, and (b) demand for the new paradigm by business, industry, government and science was overwhelming.

The IP is unlikely to have a comparably quiet and smooth ride into acceptance as economics. The IP has already failed the first condition for quiet and smooth acceptance as economics. The IP provides a parsimonious, unified explanation of the time-series of statistics of labour income inequality in the US over the last half century that obsolesces a large speculative literature in US labour economics on those time-series (Angle, 2005, 2006a, 2007a). The IP is ignored in the economics literature and rejected by economics journals. If the IP were accepted as economics by economists, it would contradict a great many papers published by economists on time-series of inequality statistics of US labour income. Some of these papers are only descriptive. Most, however, cannot resist the temptation to offer a speculative, even fanciful, explanation of what they describe (for example, a nonexistent emerging bimodality of the US distribution of labour income; cf. Levy and Murnane, 1992). So the second condition, demand for applications of the IP, is the more likely way that the IP may succeed in being adopted as economics. But at present the IP also fails the second condition because it has, as yet, no application used by business, industry, or government. So, given the IP's present failure to meet the two conditions that facilitated the acceptance of the statistical learning paradigm into the academic discipline of statistics in the US, it looks as if acceptance of the IP as economics by economists will only occur in a distant future and only if useful applications of the IP are found and demand for them diverts resources away from traditional economic applications.

As a first step toward that future, this article proposes an application of the IP, to estimating personal income distributions in small areas, perhaps an application of interest to consumer market researchers who locate stores near customers, particularly 'up market' customers, people with large incomes. There is some tentative evidence of the usefulness of the IP in this regard. Although some PhD's in economics may do small area estimates of personal income for consumer market research firms, it is not a core function of PhD economists. So even if the IP proves useful to the consumer market research industry, that fact alone would not induce economists to accept the IP in the way that demand for computationally intensive data analysis induced the discipline of statistics to accept it as a new paradigm.

Appendix 1. The Inequality Process as Econophysics

Chakrabarti, Bikas K., Anirban Chakraborti, Satya R. Chakravarty, and Arnab Chatterjee	2013	*Econophysics of Income and Wealth Distributions*	Cambridge, UK: Cambridge University Press
Patriarca, Marco and Anirban Chakraborti	2013	'Kinetic Exchange Models: From Molecular Physics to Social Science'	arXiv:1305.0768v2 [physics.soc-ph] 11 May 2013

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Kang Liu, N. Lubbers, W. Klein, J. Tobochnik, B. Boghosian, and Harvey Gould	2013	'The Effect of Growth on Equality in Models of the Economy'	arXiv:1305.0794v1 [q-fin.GN] 3 May 2013
Lopez-Ruiz Ricardo and Sanudo Jaime	2012	'Geometrical Derivation of Equilibrium Distributions in Some Stochastic Systems'	arXiv:1208.0207v1 [nlin.AO] 1 Aug 2012
Bustos-Guajardo, R. and Cristian F. Moukarzel	2012	'Yard-Sale Exchange on Networks: Wealth Sharing and Wealth Appropriation'	arXiv: 1208.4409v1 [q-fin.GN] 22 Aug 2012
Toscani, Giuseppe, Carlo Brugna and Stefano Demichelis	2012	'Kinetic Models for the Trading of Goods'	arXiv:1208.6305v1 [q-fin.GN] 29 Aug 2012
Fellingham, N., F. V. Kusmartsev	2011	A Study of the Distribution of Wealth in a Stochastic Non-Markovian Market	Hyperion International Journal of Econophysics & New Economy 4 (2) 245–271.
Ferrero, J. C.	2011	A Statistical Analysis of Stratification and Inequity in the Income Distribution	European Physics Journal B 80, 255–261 (2011) DOI: 10.1140/epjb/e2011-11018-2
Chen, Shu-Heng and Sai-Ping Li.	2011	'Econophysics: Bridges over a Turbulent Current'	*International Review of Financial Analysis* (July, 2011): doi:10.1016/j.irfa.2011.07.001
Kuerten, K. E. and F. V. Kusmartsev	2011	'Bose-Einstein Distribution of Money in a Free-market Economy II'	*Europhysics Letters* 93, #2, 28003. DOI: 10.1209/0295-5075/93*28003*
Matsuo, M. Y.	2011	'Hierarchical Mechanism of Development of Wealth and Structure for a Pre-modern Local Society'	*Physical Review E* 83, 066110. http://pre.aps.org/abstract/PRE/v83/16/e066110
Sinha, Sitabhra, Arnab Chatterjee, Anirban Chakraborti, and Bikas Chakrabarti	2011	*Econophysics: An Introduction*	Weinheim, Germany: Wiley-VCH
Bassetti, F. and G. Toscani	2010	'Explicit Equilibria in a Kinetic Energy Model of Gambling'	*Physical Review E* 83, 066115
Chakrabarti, Anindya and Bikas Chakrabarti	2010	'Statistical Theories of Income and Wealth Distribution'	*Economics E-journal* v. 4. [http://www.economics-ejournal.org/economics/journalarticles/2010-4
Sokolov, Andrey, Andrew Melatos, and Tien Kieu.	2010	'Laplace Transform Analysis of a Multiplicative Asset Transfer Model'	*Physica A*
Angle, John, François Nielsen, and Enrico Scalas.	2009	'The Kuznets Curve and the Inequality Process', pp. 125–138. In Banasri Basu, Bikas K. Chakrabarti, Satya R. Chakravarty, Kausik Gangopadhyay (Eds), *Econophysics and Economics of Games, Social Choices and Quantitative Techniques*	(Proceedings of the Econophys-Kolkata IV Conference, March 2009, Kolkata, India, jointly sponsored by the Indian Statistical Institute and the Saha Institute of Nuclear Physics) Milan: Springer
Chakrabarti, Anindya and Bikas Chakrabarti	2009	'Microeconomics of the Ideal Gas Like Market Models'	Physica A vol. 388 (#19) pp. 4151-4158. doi:10.1016/j.physa.2009.06.038. [online at http://arxiv.org/abs/0905.3972]
Lopez-Ruiz, Ricardo, Jaime Sanudo, and Xavier Calbet	2009	'Equiprobability, Entropy, Gamma Distributions, and Other Geometrical Questions in Multi-agent Systems'	Entropy 11, 959-971; doi:10.3390/e11040959 [online at: http://www.mdpi.com/1099-4300/11/4/959/pdf]

Yakovenko, Victor and J. Barkley Rosser Jr	2009	'Colloquium: Statistical Mechanics of Money, Wealth, and Income' Reviewing the history of particle system models of income distribution, Yakovenko and Rosser write (p. 3, pagination of on-line version): 'Actually, this approach was pioneered by the sociologist John Angle (1986, 1992, 1993, 1996, 2002) already in the 1980s. However, his work was largely unknown until it was brought to the attention of econophysicists by the economist Thomas Lux (2005). Now, Angle's work is widely cited in econophysics literature (Angle, 2006). Meanwhile, the physicists Ispolatov, Krapivsky and Redner (1998) independently introduced a statistical model of pairwise money transfer between economic agents, which is equivalent to the model of Angle.'	*Reviews of Modern Physics* 81, 1703–1725 [online at http://arxiv.org/abs/0905.1518]
Düring, Bertram, Daniel Matthes, Giuseppe Toscani	2008	'A Boltzmann-type Approach to the Formation of Wealth Distribution Curves'	Institute for Analysis and Scientific Computing, Vienna University Technology
Bassitti, Frederico, Lucia Ladelli, and Daniel Matthes	2008	'Central Limit Theorem for a Class of One-dimensional Equations'	Institute for Analysis and Scientific Computing, Vienna University of Technology, Report #37 ISBN 978-3-902627-01-8
Chakraborti, Anirban, and Marco Patriarca	2008	'Gamma-distribution and Wealth Inequality'	Pramana [Indian Academy of Sciences] 71(#2), 233–243
Hayes, Brian	2008	Group Theory in the Bedroom and Other Mathematical Diversions; New York: Hill and Wang (division of Farrar, Straus, and Giroux)	New York: Hill and Wang (division of Farrar, Straus, and Giroux)
Lux, Thomas	2008	'Applications of Statistical Physics in Economics and Finance'	In J. Barkley Rosser Jr (Ed.), *Handbook of Research on Complexity*. London: Edward Elgar.
Garibaldi, U. E. Scalas, and P. Viarengo	2007	Statistical equilibrium in simple exchange games II The redistribution game	*European Physics Journal B*, 60, 241–246 (2007) DOI: 10.1140/epjb/e2007-00338-5
Angle, John	2007	'The Macro Model of the Inequality Process and The Surging Relative Frequency of Large Wage Incomes'	Pp. 171–196 in A. Chatterjee and B.K. Chakrabarti (Eds), *The Econophysics of Markets and Networks.* (Proceedings of the Econophys-Kolkata III Conference, March 2007. Milan: Springer [ISBN: 978-8847006645]. [online at: http://arxiv.org/abs/0705.3430]
Chatterjee, Arnab, Sitabhra Sinha, Bikas Chakrabarti	2007	'Economic Inequality: Is It Natural?'	Current Science 92: (#10, May 25, 2007): 1383-1389. [online at: http://www.ias.ac.in/currsci/May252007/1383.pdf]
Yakovenko, Victor	2007	'Econophysics, Statistical Mechanical Approaches to'	Encyclopedia of Complexity and System Science. New York: Springer. http://refworks.springer.com/complexity]. [online at http://arxiv.org/abs/0709.3662]

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Angle, John	2006	'The Inequality Process as a Wealth Maximizing Algorithm'	*Physica A: Statistical Mechanics and Its Applications* 367:388-414. (DOI information: 10.1016/j.physa.2005.11.017). A draft version of the paper can be downloaded from the Luxembourg Income Study website: [http://www.lisproject.org/publications].
Chatterjee, Arnab and B. K. Chakrabarti	2006	Abstract of 'Heterogeneous Agents are Responsible for the Pareto Tail'.	Applications of Physics to Financial Analysis 5 (June, 2006), 7 [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf]
Foscari, Piero	2006	'Stochastic and deterministic Simulation Techniques for Traffic and Economics'	PhD dissertation, University of Ferrara, Italy. [online at http://eprints.unife.it/27/1/Foscari_2006_PhD_Thesis.pdf]
Gupta, Abhijit Kar	2006	'Econophysics and Sociophysics: Trends and Perspectives'. In B. K. Chakrabarti, A. Chakraborti & A. Chatterjee (Eds), *Econophysics and Sociophysics*	Weinheim , Germany: Wiley VCH
Richmond, Peter, Stefan Hutzler, Ricardo Coelho, and Przemek Repetowicz.	2006	'A Review of Empirical Studies and Models of Income Distributions in Society'. In B. K. Chakrabarti, A. Chakraborti & A. Chatterjee (Eds), *Econophysics and Sociophysics*	Weinheim, Germany: Wiley VCH
Scalas, Enrico, U. Garibaldi and S. Donadio	2006	'Statistical Equilibrium in Simple Exchange Games I: Methods of Solution and Application to the Bennati-Dragulescu-Yakovenko (BDY) Game'	*The European Physical Journal B* 53 (#2, September): 267-272
Chatterjee, Arnab and B. K. Chakrabarti	2006	Abstract of 'Heterogeneous Agents Are Responsible for the Pareto Tail'	Applications of Physics to Financial Analysis 5 (June, 2006), 7 [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf]
Scalas, Enrico	2006	Abstract of 'Growth and Inequality Processes: Equilibrium and Nonequilibrium Models in Physics and Economics'	Applications of Physics to Financial Analysis 5 (June, 2006), 43 [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf]
Angle, John	2006	A comment on Gallegati *et al.*'s 'Worrying Trends in Econophysics'	Pp. 250-253 in A. Chatterjee and B.K. Chakrabarti (Eds), *The Econophysics of Stocks and Other Markets* (Proceedings of the Econophys Kolkata II Conference, February, 2006). Milan: Springer.
Lux, Thomas	2005	'Emergent Statistical Wealth Distributions in Simple Monetary Exchange Models: A Critical Review'	Pp. 51-60 in A. Chatterjee, S. Yarlagadda & B.K. Chakrabarti (Eds), Econophysics of Wealth Distributions (the proceedings volume of the International Workshop on the Econophysics of Wealth Distributions, March, 2005, Kolkata, India). Milan: Springer. [online version of paper at: [http://arxiv.org/abs/cs/0506092]

Source: Developed by the author.

Appendix 2. The Empirical Phenomena that the Inequality Process Explains

1. The universal pairing (all times, all places, all cultures, all races) of the appearance of extreme social inequality (the chiefdom, society of the god-king) and concentration of wealth after egalitarian hunter/gatherers acquire a storable food surplus (Angle, 1983, 1986).
2. The pattern of the Gini concentration ratio of personal wealth and income over the course of techno-cultural evolution beyond the chiefdom (Angle, 1983, 1986).
3. The right skew and gently tapering right tail of all distributions of income and wealth (a broad statement of the Pareto Law of income and wealth distribution) (Angle, 1983, 1986).
4. (a) The sequence of shapes of the distribution of labour income by level of worker education, (b) why this sequence of shapes changes little over decades, and (c) why a gamma pdf model works well for fitting the distribution of labour income (Angle, 1990, 2002, 2003, 2006, 2007b).
5. How the unconditional distribution of personal income appears to be gamma distributed at the national level and in successively smaller regions although the gamma distribution is not closed under mixture, i.e., under aggregation by area (Angle, 1996).
6. Why the sequences of Gini concentration ratios of labour income by level of education from low to high recapitulates the sequence of Gini concentration ratios of labour income over the course of techno-cultural evolution (a social science analogue of 'ontogeny repeats phylogeny') (Angle, 1983, 1986, 2002a, 2003a, 2006a, 2006b, 2007b).
7. Why the sequence of shapes of the distribution of labour income by level of education from low to high recapitulates the sequence of shapes of the distribution of labour income over the course techno-cultural evolution (a social science analogue of 'ontogeny repeats phylogeny') (Angle, 1983, 1986, 2002a, 2003a, 2006a, 2006b, 2007b).
8. The old saw, 'A rising tide lifts all boats.' to express view that most workers regardless of size of earnings benefit from a business expansion, if modified to 'A rising tide lifts the logarithm of all boats equally.' (Angle, 2006a, 2007a)
9. The dynamics of the distribution of labour income conditioned on education as a function of the unconditional mean of labour income and the distribution of education in the labour force (Angle, 2003a, 2006a, 2006b, 2007b).
10. The pattern of correlations of the relative frequency of an income smaller than the mean with relative frequencies of other income amounts (Angle, 2005, 2006a).
11. The surge in the relative frequency of large incomes in a business expansion (Angle, 2007b).
12. The right tail of income and wealth distributions being heavy enough to account for total annual wage and salary income in the U.S. National Income and Product Accounts (Angle, 2001).
13. Why and how the distribution of labour income is different from the distribution of income from tangible assets; (Angle, 1997).
14. Why the IP's parameters estimated from certain statistics of the year to year labour incomes of individual workers are ordered as predicted by the IP's meta-theory and approximate estimates of the same parameters from the fit of the IP's stationary distribution to the distribution of wage income conditioned on education (Angle, 2002a).
15. The Kuznets Curve of the Gini concentration ratio of labour income during the industrialization of an agrarian economy (Angle, Nielsen & Scalas, 2009).
16. In an elaboration of the basic IP: if a particle in a coalition of particles has a probability different from 50 per cent of winning a competitive encounter with a particle not in the coalition, this modified IP reproduces features of the joint distribution of personal income to African-Americans and other Americans:
    1. the smaller median personal income of African-Americans than other Americans;
    2. the difference in shapes between the African-American distribution of personal income and that of other Americans; this difference corresponds to a larger Gini concentration of the African American distribution;
    3. the percentage minority effect on discrimination (the larger the minority, the more severe discrimination on a per capita basis, as reflected in a bigger difference between the median personal incomes of African-Americans and other Americans in areas with a larger per cent African-American);
    4. the high ratio of median African-American personal income to the median of other Americans in areas where the Gini concentration ratio of the personal income of other Americans is low;
    5. the high ratio of median African-American to that of other Americans in areas where the median income of other Americans is high;
    6. the fact that relationships in four and five can be reduced in magnitude by controlling for a measure of economic development of an area or percentage African-American;
    7. the greater hostility of poorer other Americans to African-Americans than wealthier other Americans (Angle, 1992).

Source: Developed by the author.

Appendix 3. The Inequality Process (IP) Puts Accepted Propositions of Mainstream Economics on a Firm Scientific Footing

Widely Accepted Proposition in Economics	Inequality Process' Explanation
1. All distributions of labour income are right skewed with tapering right tails; hence the impossibility of radical egalitarianism, the inference motivating Pareto's study of income and wealth distribution.	The IP generates right skewed distributions shaped like empirical distributions of labour income or personal assets (depending on the value of the particle parameter).
2. Differences of wealth and income arise easily, naturally, and inevitably via a ubiquitous stochastic process; cf. the most general statement of Gibrat's Law; hence the impossibility of radical egalitarianism.	In the IP, differences of wealth arise easily, naturally, and inevitably, via a ubiquitous stochastic process.
3. A worker's earnings are tied to that worker's productivity [i.e., a central tenet of economics since Aesop's fable of the ant and the grasshopper was all there was to economics] but there is a wide distribution of returns to similarly productive workers.	In the IP's Macro Model, an approximation to its stationary distribution, a particle's expected wealth is determined by the ratio of mean productivity in the population to that of an individual. There is a distribution of wealth around this expectation.
4. Labour incomes small and large benefit from a business expansion strong enough to increase mean labour income, i.e., there is a community of interest between all workers regardless of their earnings in a business expansion. A conclusion encapsulated in the saying, 'A rising tide lifts all boats'.	In the IP's Macro Model, an increase in the unconditional mean of wealth increases all percentiles of the stationary distribution of wealth by an equal factor. In pithy statement form: 'A rising tide lifts the logarithm of all boats equally.'
5. Competition transfers wealth to the more productive of wealth via transactions without central direction, that is, via parallel processing.	In the IP, competition between particles causes wealth to flow via transactions from particles that are by hypothesis and empirical analogue less productive of wealth to those that are more productive of wealth, enabling the more productive to create more wealth, explaining economic growth without a) requiring knowledge of how wealth is produced or b) central direction, i.e., with a minimum of information, two reasons why the IP may have been naturally selected. These features enable the IP to operate homogeneously over the entire course of techno-cultural evolution independently of wealth level.
6. Competition and transactions maximize societal gross product and over the long run drive techno-cultural evolution.	The Inequality Process operates as an evolutionary wealth maximiser in the whole population of particles, given a relaxation of the zero-sum constraint on wealth transfers within the model, by transferring wealth to the more productive.

Source: Developed by the author.

Appendix 4: The Specification of the Inequality Process (IP) from a Verbal Cornerstone of Economic Anthropology

1. The Specification of the Model

The Inequality Process (IP) is a mathematical model specified from the Surplus Theory of Social Stratification, an old theory of economic anthropology that explains why the first appearance of great inequality of wealth in the archaeological record of a population appears in the same layer as the first appearance of abundant stored food (Childe, 1944; Dalton, 1960, 1963; Harris, 1959; Herskovits, 1940). This archaeological layer corresponds to the transition of a population that previously lived as hunter-gatherers, with few differences of wealth and no ascribed ruling clan, into the inegalitarian chiefdom, the society of the god-king. This transition was apparently universal: all times, all places, all cultures, all races. The Surplus Theory offers an elegantly simple explanation: (a) there is widespread competition in all human groups, (b) hunter-gatherers mostly live from hand to mouth, but (c) when because of a richer ecological niche or the acquisition of agricultural technologies, the hunting and gathering population acquires an abundance of storable food, the competition that existed all along in the group concentrates control of stored abundance in few hands.

While the Surplus Theory is an elegant verbal explanation of the universality of the transformation of the societal form anthropologists view as the most egalitarian, the hunter/gatherer, into the societal form they see as the most inegalitarian, the chiefdom, the Surplus Theory has no explanation for why further techno-cultural evolution beyond the chiefdom led to less concentration of wealth than in the chiefdom. Gerhard Lenski (1966) proposed a number of speculative amendments to the Surplus Theory to account for the decreasing trend in the concentration of wealth over the course of techno-cultural evolution from the chiefdom on. The IP is specified from one of Lenski's speculations: Worker skills are a valuable capital good that workers can easily withhold in bargaining for a larger share of the wealth they create, and consequently, a greater share of the wealth produced by advancing technology is retained by workers whose knowledge and skills embody that technology. Worker skill, human capital, becomes a larger fraction of aggregate societal wealth as populations attain a higher level of technological evolution.

The IP is abstracted from the Surplus Theory of Social Stratification as modified by Gerhard Lenski with the help of the principle of parsimony. The specification of a model from verbal theory is an art. In the specification of the IP the simplest model of competition was sought consistent with the verbal meta-theory. The model is a particle system. Its entities represent people but are so simple, they qualify as particles. The IP's particles have only two characteristics, one transient, one semi-permanent. The transient characteristic is wealth; it changes with every competitive encounter with another particle. The semi-permanent characteristic is the fraction of wealth the particle gives up when it loses an encounter. It is semi-permanent in the way a worker's skill level is semi-permanent. Competitive encounters are pairwise because (a) pairwise is simplest, (b) verbal theory offers no guidance on the organization of the extraction of surplus wealth from workers, and (c) competition in groups, regardless of size or composition, that transfers wealth between people results in a net gain or loss for each person—just as in binary competition. Competition is zero sum in the IP because of its simplicity: no model of wealth production or consumption. Lenski treats per capita economic product as a function of technology, making no effort to create a theory of wealth production over the techno-cultural spectrum.

2. The Equations of the Inequality Process (IP)

The IP is defined by the equations for the transfer of wealth between particles in a competitive encounter, the 'transition equations':

$$x_{it} = x_{i(t-1)} + d_t\, \omega_{\theta j}\, x_{j(t-1)} - (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)}$$
$$x_{jt} = x_{j(t-1)} + d_t\, \omega_{\theta j}\, x_{j(t-1)} + (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)}$$

where:

$x_{it} \equiv$ *particle i's wealth at time – step t*

$x_{j(t-1)} \equiv$ *particle j's wealth at time – step* $(t-1)$

$0 < \omega_{\theta j} < 1.0$ fraction lost in loss by particle j

$0 < \omega_{\psi i} < 1.0$ fraction lost in loss by particle i

d_t = *an i.i.d.* 0,1 *uniform discrete r.v. at time–step t*

The IP generates a stationary distribution of wealth in each ω_ψ equivalence class of particle that is approximately, but not exactly, a gamma probability density function (pdf). The IP's unconditional stationary distribution of wealth is thus approximately a mixture of gamma pdf's with different shape and scale parameters. Since the IP was first published in 1983, several related particle system models of personal income and wealth have been published (for example, Chakraborti &Chakrabarti, 2000; Dragulescu & Yakovenko, 2000). The differences between these and the Inequality Process are discussed in Angle (2012).

The stationary distribution of the Inequality process (IP) can be approximated by a gamma pdf. The Macro Model of the Inequality Process (MMIP) is the approximating gamma pdf with shape and scale parameters expressed in terms of a particular value of the particle parameter, ω_ψ, and the harmonic mean of all the ω_ψ's, $\tilde{\omega}_t$.

$$f(x_\psi) \equiv \frac{\lambda_{\psi t}^{\alpha_\psi}}{\Gamma(\alpha_\psi)}\, x_\psi^{\alpha_\psi - 1}\, e^{-\lambda_{\psi t} x_\psi}$$

$x_\psi \equiv$ wealth in the ω_ψ equivalence class in multiples of μ_t

$x_\psi > 0$

$$\alpha_\psi \equiv \textit{shape parameter} \approx \frac{1-\omega_\psi}{\omega_\psi}$$

$$\lambda_{\psi t} \equiv \textit{scale parameter} \approx \frac{1-\omega_\psi}{\tilde{\omega}_t\, \mu_t}$$

$\tilde{\omega}_t \equiv$ *harmonic mean of* $\omega_\psi' s$

Given the expression for the mean of a random variable in the two parameter gamma pdf, the MMIP's estimator of the mean of particle wealth, x_ψ, in the ω_ψ equivalence class is, $\mu_{\psi t}$, is:

$$\mu_{\psi t} = \frac{\alpha_\psi}{\lambda_{\psi t}} \approx \frac{(\tilde{\omega}_t\, \mu_t)}{\omega_\psi}$$

where μ_t is the unconditional mean of wealth. See Salem and Mount (1974) for an approximation formula for the median of a gamma pdf.

3. The Dynamics of the Macro Model of the Inequality Process (MMIP)

The dynamics of the MMIP in each ω_ψ equivalence class are entirely exogenous. They are driven by the unconditional mean of wealth, μ_t, and the distribution of workers by level of education in the labour force as reflected in the harmonic mean of the ω_ψ's and are expressed solely in terms of the scale parameter, $\lambda_{\psi t}$. The shape of the stationary distribution of particles in the ω_ψ equivalence class does not change.

The MMIP's model of the distribution of wealth is stretched to the right (over larger wealth (x) amounts), or compressed to the left (over smaller wealth amounts) according to whether the product $(\tilde{\omega}_t\, \mu_t)$ increases (stretches distribution to the right) or decreases (compresses distribution to the left.

When the MMIP is fitted to the distribution of annual wage and salary income conditioned on education (using education as the available indicator of worker skill) in the US from 1961 on, the MMIP provides a good fit (Angle, 1997, 1998, 1999a,b, 2001, 2002b,c, 2003a,c, 2005, 2006a,b, 2007a, 2009, 2012). ω_ψ varies inversely with worker education level as expected under the IP's meta-theory. The dynamics of the US distribution of annual wage and salary income conditioned on education are in the scale of the distribution driven by two exogenous components, the unconditional mean of annual wage and salary income and the education level of the workers, measured by the harmonic mean of the ω_ψ's, $\tilde{\omega}_t$. As education levels of workers in the US rose, the estimated $\tilde{\omega}_t$ fell, as implied by the IP's meta-theory. The two components of the product $(\tilde{\omega}_t\ \mu_t)$ drive the dynamics of the MMIP and the distribution of labour income in opposite directions.

Taking the partial derivative of the MMIP with respect to the driver of its dynamics, $(\tilde{\omega}_t\ \mu_t)$, gives an expression for the dynamics of the MMIP and the distribution of labour income:

$$\frac{\partial f_{\psi t}(x_0)}{\partial(\tilde{\omega}_t \mu_t)} = f_{\psi t}(x_0)\lambda_{\psi t}\left(\frac{x_0-\mu_{\psi t}}{\tilde{\omega}_t \mu_t}\right)$$

$$= f_{\psi t}(x_0)\,\frac{(1-\omega_\psi)}{(\tilde{\omega}_t\ \mu_t)^2}\ (x_0-\mu_{\psi t})$$

where x_0 is an arbitrary income or wealth amount. This equation implies a great surge in the number of very large incomes when $(\tilde{\omega}_t\ \mu_t)$ increases. This prediction has been confirmed with US data (Angle, 2007a). Given India's rapidly rising mean personal income and levels of education in its labour force, this equation may be of especial interest to Indian consumer market research firms.

Source: Developed by the author.

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