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**APPLICATION OF NEURAL NETWORK IN THE PREDICTION  
OF SELF-OSCILLATIONS AND SIGNAL STABILIZATION IN NON-LINEAR  
MULTIVARIABLE SYSTEMS**

Dr.K.C.Patra and A.K.Swain  
Indira Gandhi Institute of Technology, Sarang, Talcher, India

S.Majhi

**ABSTRACT:**

*In this paper attention is directed to the systematic development of possible structure of nonlinear multivariable system for the prediction of self-oscillations and signal stabilization. The generalization of the technique along with the application of Neural Network provides explicit and novel insight into the problem of greater complexity. The technique so developed has been illustrated through example and its accuracy has been substantiated comparing with the results from digital simulation.*

**1: Introduction**

The importance of system structure in the development of the technique most suited for analysis of nonlinear self-oscillations (limit cycles) and signal stabilization has been implicitly recognized in the literature concerned with two and higher dimensional nonlinear systems [1, 2, 7]. Signal stabilization of single input and single output systems incorporating only one nonlinear element has been extensively investigated by many researchers [3, 4, 6]. Recently, Neural Networks, a novel modelling technique captured the attention of many researchers for partially or fully unknown nonlinear and ill-defined plant [8].

In this paper the nature of possible structure of nonlinear multivariable system has been examined and subsequently a most general structure has been developed. The use of high frequency sinusoidal dither for signal stabilization of two dimensional nonlinear system has been determined. This paper deals with the neural modelling of a two dimensional nonlinear system and subsequently illustrates through example the potential of neural networks for limit cycle prediction and signal stabilization which has been compared with the results of digital simulation.

**2: Structure of a General Multivariable Nonlinear System**

Consider a nonlinear system incorporating an arbitrary number of nonlinear elements that can be isolated from the associated linear elements [6]. Such a system is schematically represented in Fig.1(a), where  $x$  is a 'l' vector,  $y$  is a 'm' vector and  $N$  is a 'm x l' matrix nonlinear function. The system input  $u$  and output  $c$  are vectors of dimension 'k' and 'n' respectively. In addition to the multivariable nonlinear element  $N$ , the system may incorporate any number of arbitrary linear matrix elements of appropriate dimensions connected in an arbitrary manner. Fig.1(b) shows a multidimensional signal flow graph representing the most general situation wherein the signal  $x$  can be derived from the vectors  $u, y$  and  $c$  passing through appropriate linear transfer function matrices of dimensions 'l x k', 'l x m' and 'l x n' respectively.

Similarly the output vectors  $u$ ,  $y$  and  $x$  through linear matrix transmittances of dimension ' $n \times k$ ', ' $n \times m$ ' and ' $n \times l$ ' respectively shown in Fig.1(c). Consequently, the signal flow graph of the most general representation of the structure of a system belonging to the class under consideration is obtained as the union of the Figs.1(b) and 1(c). This is shown in Fig.1(d) and a simplified equivalent signal flow graph is shown in Fig.1(e) with its equivalent block diagram representation in Fig.2. The variables in Fig.2 are related by the equations:

$$y = y(x); \quad x = G_1 u - H G_2 y; \quad c = G_2 y + G_3 u;$$

where,  $u$ ,  $x$ ,  $y$  and  $c$  are vectors of dimension  $k, l, m$  and  $n$  respectively and  $G_1, G_2, G_3$  and  $H$  are linear transfer function matrices of dimensions ' $l \times k$ ', ' $n \times m$ ', ' $n \times k$ ' and ' $l \times m$ ' respectively.

### 3: Analysis of Self Oscillations and Signal Stabilization

For the autonomous state ( $[U] = [0]$ ), the system of Fig.2 can be equivalently represented as shown in Fig.3. Making use of Describing function approximation of nonlinear elements, the matrix equation for the system of Fig.3 can be expressed as:

$$X = -HGN(x)X = AX \dots\dots\dots(1)$$

where,  $A = -HGN(x)$ . Visualizing Eq.1 as a transformation of the vector  $X$  onto itself, we note that for a limit cycle to exist, the following two conditions must be satisfied:

- (i) For every nontrivial solution of  $X$ , the matrix  $A$  must have an eigen value  $\lambda$  equal to unity
- and, (ii) The eigen vector of  $A$  corresponding to this unity eigen value must be coincident with  $X$ .

When the system is exhibiting limit cycles the possibility of quenching limit cycles by injecting a high frequency dither signal (signal stabilization) can also be examined using the above two conditions and replacing  $N$  by respective equivalent gains[4]. During the process of signal stabilization the phenomena of complex oscillations leading to synchronization and desynchronizations are examined making use of Dual Input Describing Functions(DIDF) and Incremental Describing Functions(IDF) of the nonlinear elements respectively.

As in Fig.5, the modeling of a two dimensional nonlinear system for prediction of limit cycles and signal stabilization with two separate neural networks has considered. In general for an  $n$ -dimensional system,  $n$  separate neural network blocks are required. The neural network used here is a feed forward network with Kalman filter algorithm for weight updation during learning phase. The mean square error is minimized with respect to the summation outputs[5], in contrast to the standard conventional backpropagation algorithm, which is with respect to the weights. The following steps highlight the weight updation scheme for  $j$ th layer of the network,

The Kalman gain vector:

$$k_j(t) = [R_j^{-1}(t-1) x_{j-1}(t)] / [b_j + x_{j-1}^T(t) R_j^{-1}(t-1) x_{j-1}(t)]$$

where,  $R$  is the correlation matrix of the training set,  $x$  is the

input vector and  $b$  is the forgetting factor.  
 The update equation:

$$R_j^{-1}(t) = [R_j^{-1}(t-1) - k_j(t)x_{j-1}^T R_j^{-1}(t-1)]b_j^{-1}$$

The forgetting factor updation:  $b_j = b_0 b_j + (1 - b_0)$   
 The weight update equation:

$$\text{Output layer: } w_{Lk}(t) = w_{Lk}(t-1) + k_L(t)(d_k - Y_k)$$

$$\text{Hidden layer: } w_{jk}(t) = w_{jk}(t-1) + k_j(t)e_{jk}(t)\mu_j$$

where,  $d$  is the desired summation output,  $L$  is the output layer,  $y$  is the calculated output,  $e$  is the error and,  $\mu$  is the backpropagation step size.

#### Example

Consider the system shown in Fig.4 with  $G_1(s) = 2/s(s+1)^2$ ;  $G_2(s) = 1/s(s+4)$ ; and the two nonlinear elements having ideal relay characteristics with  $M_1=1.0$  and  $M_2=1.126$ . Interpreting the system of Fig.4 in the form of Fig.3 and employing the condition (i), i.e.,  $\lambda=1$ , we get,

$$1 + N_1 G_1 + N_2 G_2 + 2N_1 N_2 G_1 G_2 = 0 \dots \dots \dots (2)$$

Condition (ii) in conjunction with Eqn.1 leads to

$$X_1/X_2 = |N_2 G_2|/|1+N_1 G_1| \text{ or } X_1/X_2 = |1+N_2 G_2|/|N_1 G_1| \dots \dots (3)$$

Eqn.3 and two equations obtained by separating the real and imaginary parts of Eqn.2 are used in solving three unknowns such as the frequency of self oscillations( $\omega$ ), amplitude of self oscillations( $X_1$  and  $X_2$  or  $C_1$  and  $C_2$ ).

The high frequency signal  $B \sin \omega_c t$  is injected at  $u_1$  [cf Fig.4] and the signal stabilization is examined. The variables of complex oscillations are obtained using Eqns.2 and 3 but replacing  $N_1$  by  $N_{1 \text{ req}}$  (DIDF). The critical value of  $B$  for desynchronization is obtained in the same manner but replacing  $N_1$  and  $N_2$  by respective IDFs.

The training patterns required for the training of neural network for our simulation study has been obtained from the state variable analysis of the plant. The network used here is a two layer feed forward network with 16 inputs, 20 hidden layer neurons and 16 output layer neurons, the backpropagation step size  $\mu$  is 30, the slope of the tangent hyperbolic function is 0.1, initial forgetting factor is 0.9 and forgetting factor is 0.98, number of iterations is 1500. The results of the example for selfoscillations, and signal stabilization obtained from analytical computation, neural network and digital simulation are shown in table and Fig.6 respectively.

#### 4. Conclusion

In the process of signal stabilization two important limiting phenomena, namely synchronization and desynchronization in the class of systems under consideration have been investigated and the technique for analyzing the phenomena has been presented. The method of analysis aids the conceptual visualization of the

mechanism leading to these interesting phenomena occurred in the real time process. The application of neural network based on the above concept developed through fast backpropagation learning algorithm can give better results in real time processing of the multivariable systems. Comparison of results of analysis with the results from digital simulation and neural computation show that the use of the technique in the analysis lead to results with an acceptable accuracy.

**References:**

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Table:RESULTS OF THE EXAMPLE

Methods of computation	$\omega$ in rad/sec	$X_1$	$X_2$
Analytical	0.8	2.0	1.98
Digital Simulation	0.791	2.05	2.04
Neural Net	0.789	2.045	2.038

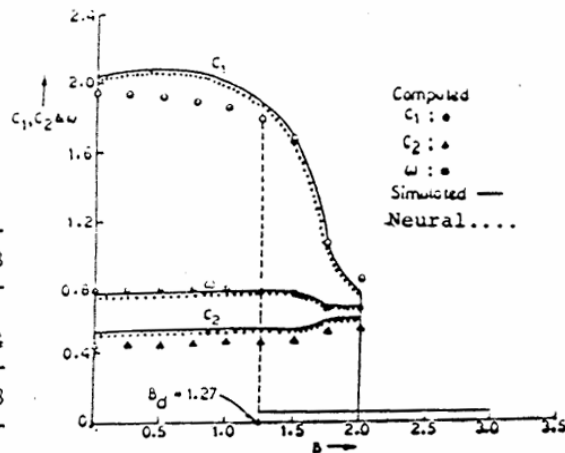


FIG. 6 : RESULTS OF EXAMPLE FOR ILLUSTRATING COMPLEX OSCILLATIONS

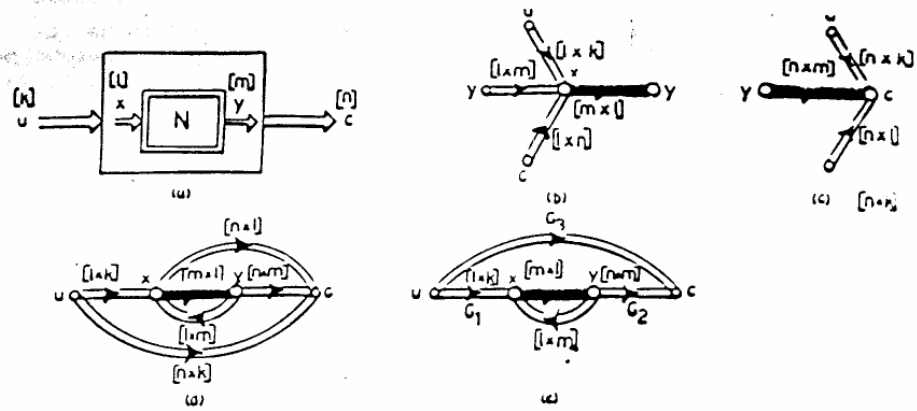


Fig.1.S F G BASED DEVELOPMENT OF GENERAL STRUCTURE FOR THE NONLINEAR MULTIVARIABLE SYSTEM.

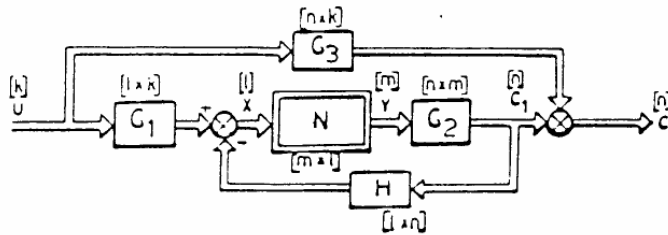


Fig.2.BLOCK DIAGRAM REPRESENTATION OF A MOST GENERAL NONLINEAR MULTIVARIABLE SYSTEM.

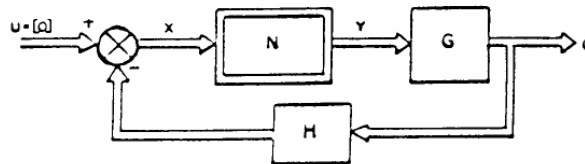


Fig.3.EQUIVALENT OF THE SYSTEM OF Fig.2 FOR  $U=[0]$ .

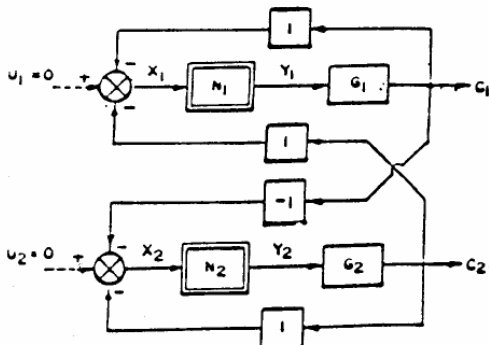


Fig.4.BLOCK DIAGRAM REPRESENTATION OF THE SYSTEM CONSIDERED IN THE EXAMPLE.

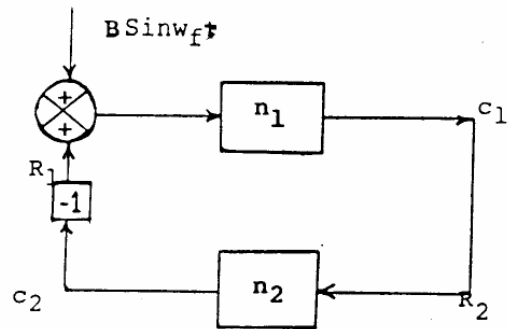


Fig.5.NEURAL MODEL FOR THE SYSTEM OF Fig.4.